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A Hybrid Representation of Urban Traffic Networks using Multi-agent Systems and Petri Nets

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Abstract— **This paper presents a novel modeling of complex urban traffic networks based on the interconnections of agents which represent the vehicular fluid dynamics in physical spaces like streets and intersections, supervised by a Petri Net. The agent's behavior is ruled by simple nonlinear differential equations based on the vehicular density, average vehicle velocities, congestions and occupation capacities. The Petri Net is designed to manage the traffic light cycles, enabling or disabling the flow of vehicles to and from adjacent agents. The main contribution of the paper is to define the basis of a hybrid and modular system to be addressed by the theory of multi-agent systems and supervisory control of Petri Nets. An example modeling a real intersection in the city of Montevideo is presented. The model's behavior is compared with a common traffic simulator in order to show the potential of the approach and its scalability to more complex traffic networks.**

I. INTRODUCTION

A Multi-Agent System (MAS) consists of a set of dynamical autonomous agents interacting between each other to solve a complex task through collaboration. This approach has been used in many applications from engineering such as Robotics to economics and social sciences [1]. A robotic system is often seen as a group of mobile hardware agents where a complex task requires a precise motion coordination strategy. Centralized and decentralized strategies are possible implying a higher level supervisor could be present to help the coordination tasks. This work focuses on applying a supervised MAS in the context of urban traffic systems. A review of multi-agent based strategies for transportation systems is presented in [2].

The growth in the world's population and vehicles have generated problems in traffic systems, such as congestion, pollution [3] [4], and social [5], especially in urban areas. For example, Mexico City have 20.4 million inhabitants which requires strategies to improve vehicular traffic within an accelerated urban growth [6]. Approximately, $1,200,000$ cars circulate daily, and 22 million trips are made in the metropolitan area, mostly to downtown, according to the reports of the University Study Program for Mexico City [7]. The Secretary of Telecommunications and Transport establishes all the regulations for standardization of intersections, traffic lights, velocity limits, etc. to manage the network behavior.

To improve a transportation system involves the modeling of vehicle flows through the network and development of control algorithms that optimize them, mainly during peak hours. They should manage traffic light cycles and splits, variable speed limits, taking into account street and intersection characteristics and more complex scenarios like road obstructions, weather conditions, etc. From the perspective of traffic control engineering, the models are classified in microscopic, mesoscopic and macroscopic [8], [9] according to its level of detail, using physical descriptions by continuous differential equations to discrete-event behaviors, in a deterministic or stochastic setting.

The microscopic level considers cars as particles with very specific features such as type of vehicle, origin-destination, speed, acceleration, and sometimes even driver behavior [10]. In the macroscopic level, the vehicular traffic is considered as a continuous flow, where no distinction is made between the vehicles moving in the streets. The main variables are street flows, densities and velocities, as well as the relationships between them. State space models are created in order to propose predictive and adaptive models [11]. Finally, the mesoscopic level falls between the microscopic and macroscopic models where traffic flows are represented in groups with private properties and parameters such as density and speed [12].

Petri Nets (PN) have also been widely used in the modeling, simulation and optimization of urban traffic systems [13], [14]. Simple, timed and colored PN have been applied to describe microscopic levels of traffic, where the position of each vehicle is represented by a token in the net (e.g. [15], [16], [17], [18]). In [19] a combination of continuous and timed PNs along with an adaptive spatial discretization equation become a model for traffic flow in a macroscopic level. The approaches do not scale well with model size due to the detailed description of the streets and intersections. Therefore, the papers study only simple cases. Hybrid system approaches such as hybrid automata [20] or mixed logical dynamical (MLD) models have also been applied [21].

In order to evaluate different traffic models, several simulation platforms have been developed using graphic editors to draw, simulate and analyze the network capacity under different control strategies [22]. The software platforms incorporate many features like street capacity, number of lanes, types of cars, transfer times, flow volumes, vehicle densities, speed-density relation. Relevant platforms, according to [22], are TSIS-CORSIM (Traffic Software Integrated System,

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CORridor SIMulation), AIMSUN and SUMO (Simulation of Urban Mobility), among others.

The goal of this work is to address the modeling of a traffic network in the macroscopic level, using a multi-agent hybrid system approach to serve as the basis for future research. In this sense, this work extends the idea of MAS applied to vehicular traffic, where each segment of the network represents an agent, and the behavior of the interconnected agents is supervised by timed PN managing the traffic lights. Thus, the main contributions of this work can be summarized in the following points:

- Define a simple and modular differential equation, to describe in a macroscopic level, the street segments, intersections, vehicle sources and sinks, and any other physical space where a flow of vehicles can be defined. This model includes the effects of the traffic lights, the street and intersection saturation levels, the change of vehicle flows due to the quantity of cars and the congestion phenomena by the interconnections of the physical segments.
- To show the possibility to model different types of intersections, e.g. T and four branches, multiple branches and roundabouts using an appropriate interconnection of the models defined.
- Define PN models to represent the traffic light cycles for each type of intersection with the possibility of supervising these cycles in order to improve the flow.
- Simulate this hybrid approach, using Matlab™, in order to show the scalability of the model, establishing the core of a novel vehicular traffic simulator.

The article is organized as follows, fundamental concepts of PN are given in section II. The problem statement is presented in section III while the agent-based hybrid model description of the traffic flow is presented in section IV. The modeling example of a single intersection is established in section V. The main application to a simulation of a network in Montevideo city is given in section VI. Finally, conclusions and future work are presented in section VII.

II. Petri Net definitions

A Timed Petri-Net *TPN* extends a PN and can be defined by the six-tuple

$$TPN = (P, T, A, w, M_0, \delta), \tag{1}$$

where $P = \{p_1, p_2, \dots, p_n\}$ represents a finite set of n nodes in a graph, called *Places* and $T = \{t_1, t_2, \dots, t_m\}$ is a finite set of m nodes called *Transitions*. They are connected by a set of *arcs* $A \subseteq (P \times T) \bigcup (T \times P)$ with positive integer weights $w : A \to \mathbb{Z}^+$. M_0 is the initial marking or number of tokens contained in each place $p \in P$. Let $M_k(p) : P \to \mathbb{Z}^+$ defines the n-th vector with the number of tokens residing inside each place at time k. The k-th state or marking of the PN is achieved according to the following transition rule. A transition $t_i \in T$ is said to be enabled in a PN with finite capacity if: $M_k(p_j) \geq w(p_j, t_i), \forall p_j | w(p_j, t_i) \in w$, with $j = 1, \dots, n$ and $i = 1, \dots, m$. If t_i is enabled the next marking is defined as $M_{k+1}(p_j) = M_k(p_j) - w(p_j, t_i) + w(t_i, p_j)$.

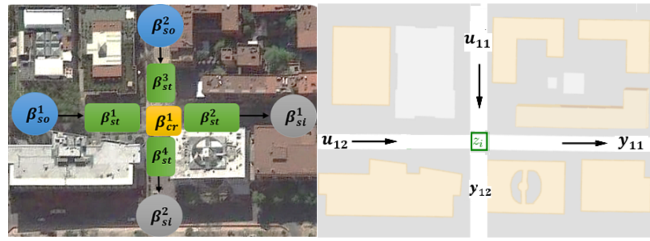

Fig. 1. Example of a simple intersection.

A *TPN* has a function $\delta : T \to \mathbb{R}^+$ that assigns a positive number τ_i to each transition t_i in the net. The term $\tau_i = \delta(t_i)$ defines the time delay between a transition t_i is enabled and the tokens are moved to their next place $M_{k+1}(p), \forall p \in P$.

III. Problem definition

As seen in Figure 1, consider $\beta = \beta_{st} \cup \beta_{cr} \cup \beta_{so} \cup \beta_{si}$ as the set of physical spaces or agents where the vehicles can be placed or move in a traffic network. $\beta_{st} = \{\beta^1_{st}, \dots, \beta^n_{st}\}$ is the set of n street agents, $\beta_{cr} = \{\beta^1_{cr}, \dots, \beta^m_{cr}\}$ is the set of m intersection agents, $\beta_{so} = \{\beta^1_{so}, \dots, \beta^k_{so}\}$ is the set of vehicle sources and $\beta_{si} = \{\beta^1_{si}, \beta^2_{si}, \dots, \beta^s_{si}\}$ is the set of car sinks. The sources that provide vehicles to the network include physical spaces like garage parkings, building parkings or new car agencies. The sinks are the agents containing cars that leave the network. Note that by the definition of sources and sinks, the total number of cars can be established as a known quantity, independently if the cars are moving in streets or intersections. Also, it helps to construct autonomous differential equations, as it will be explained below. An example of the distinct physical spaces in a real traffic network is shown in Figure 1, considering the study of a single intersection, where the inputs and outputs of cars are placed in the source and sink agents. Consider $N_i^+ \subset \beta$ as the in-adjacency set of β_i, i.e. all the physical spaces that provide cars to $\beta_i \in \beta$. Similarly, consider $N_i^- \subset \beta$ as the out-adjacency set of β_i, i.e. the physical spaces that carry out cars from $\beta_i \in \beta$. Note that it allows all the possible interconnections between street, intersections, sources and sinks. Define $z_i \in \mathbb{R}$ as the number of cars in each agent $\beta_i \in \beta$. Also, define $\alpha(z_i) \in \mathbb{R}$, $\forall \beta_i \in \beta$ as a function related to the speed of cars. This function must represent the inflow and outflow rates depending on the density of cars in the agent, which has been the focus of statistical and experimental data in papers concerned with the macroscopic level.

Problem Statement *Define a hybrid model to describe a traffic network consisting on:*

1) *A set of modular and simple flow differential equations for each z_i describing agent's occupation.*
2) *A set of functions $\alpha(z_i)$ to describe flows matching experimental data.*
3) *A set of timed Petri nets PN_i, $\forall \beta^i_{cr} \in \beta_{cr}$ to manage the traffic lights, according to the flows that arrive or leave the intersections.*

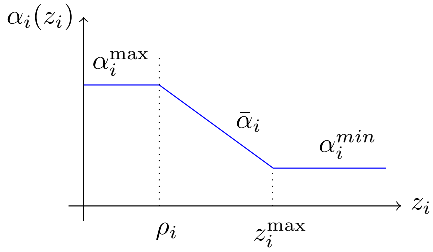

Fig. 2. Dependence of α_i with respect to z_i.

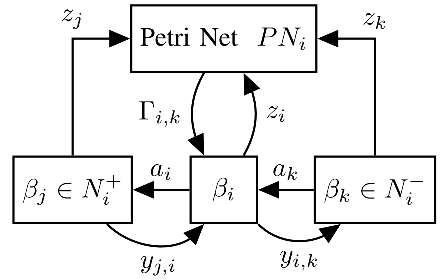

Fig. 3. Hybrid architecture proposed for a traffic network.

4) *A possible set of relationships between the Petri Net models and the differential equations to describe a feedback loop for supervisory control.*

In this hybrid scheme, the closed-loop system between the plant (differential equations) and its supervisory control (PN_i) makes it possible to study the effects of the network topology and control on the vehicle density and distribution over the network, congestions, real cycles of the traffic lights, among others.

IV. Modular equations of traffic flow

The main goal of this work is to model a traffic network in a modular way. Therefore, a differential equation representing all the physical agents in the network, i.e. streets, intersections, sources and sinks is proposed. In this way, it will be easier to build complex networks containing all intersection variants, e.g. basic T and Y intersections, four branches, multiple branches and roundabouts [23]. According to the problem definition of section III, consider a modular model of vehicle flow, for each $\beta_i \in \beta$, given by:

$$\dot{z}_i = \sum_{\beta_j \in N_i^+} u_{j,i} - \sum_{\beta_k \in N_i^-} y_{i,k} \tag{2}$$

$$y_{i,k} = \alpha_i(z_i) z_i \Gamma_{i,k} a_k(z_k), \forall \beta_k \in N_i^- \tag{3}$$

where z_i is the amount of vehicles in $\beta_i \in \beta$, u_{ji} is the input flow to agent β_i from $\beta_j \in N_i^+$ and $y_{i,k}$ is the output flow from β_i to agent $\beta_k \in N_i^-$. Note that the dynamics of z_i given by equation (2) are controlled by the input and output flows to and from agent β_i.

Equation (3) manages the vehicle discharge flow. The function $\alpha_i(z_i)$ selected in this work is shown in Figure 2, and can be expressed as

$$\alpha_i(z_i) = \begin{cases} \alpha_i^{\max}, & \text{if} \quad z_i < \rho_i \\ \bar{\alpha}_i, & \text{if} \quad \rho_i < z_i < z_i^{\max} \\ \alpha_i^{\min}, & \text{if} \quad z_i = z_i^{\max} \end{cases} \tag{4}$$

where $z_i^{\max}$ is the maximum density of cars for β_i, at ρ_i the function α_i begins to linearly decrease according to

$$\bar{\alpha}_i = \left(\frac{\alpha_i^{\min} - \alpha_i^{\max}}{z_i^{\max} - \rho_i} \right) (z_i - \rho_i) + \alpha_i^{\max} \tag{5}$$

to a minimum value $\alpha_i^{\min} < \alpha_i^{\max}$. The value of $\alpha_i^{\max}$ depends on the area of the segment β_i given by the physical length of streets, number of lanes, average vehicle size, etc. In this way the rate of discharge flow decreases when the number of vehicles in the agent β_i increases. There exist other similar functions in the literature, as presented in [24]. The output flows can be enabled or disabled by the activation functions $\Gamma_{i,k}$ and a_k. $\Gamma_{i,k}$ is defined as

$$\Gamma_{i,k} = \begin{cases} C_{i,k} M_k, & \text{if} \quad \beta_k \in \beta_{cr} \\ C_{i,k} M_i, & \text{if} \quad \beta_i \in \beta_{cr} \\ 1, & \text{otherwise} \end{cases} \tag{6}$$

where $M_k \in \mathbb{R}^{n_k \times 1}$ is the actual marking vector of the PN of n_k places representing the traffic light behaviors for intersection β_k, and $C_{i,k} \in \mathbb{R}^{1 \times n_k}$. Thus, the function $\Gamma_{i,k}$, is related to a traffic light cycle if the output $y_{i,k}$ connects the segment β_i to an intersection agent β_k supervised by a traffic light, and it is obtained as a linear combination of the vector M_k. If there exists no traffic light for β_k or $\beta_k \notin \beta_{cr}$, then the function $\Gamma_{i,k}$ is always activated, allowing a free flow of vehicles.

The functions $a_k(z_k)$ in equation (3) represent a feedback information to control the output flow of β_i according to the saturation level of the downstream segment $\beta_k \in N_i^-$. The function selected is:

$$a_k(z_k) = \frac{1}{1 + \exp\left(k_e \left(\frac{z_k}{z_k^{\max}} - \xi \right) \right)}, \tag{7}$$

where $k_e, \xi \in \mathbb{R}$. When β_k is full, $a_k \approx 0$, and the output flow of the precedent agent β_i is blocked. The equations (2)–(3) apply directly to streets and intersections, i.e. $\forall \beta_i \in \beta_{st}, \beta_{cr}$, but they can also be applied to sources $\beta_i \in \beta_{so}$ and sinks $\beta_i \in \beta_{si}$ where $N_i^+ = \emptyset$ and $N_i^- = \emptyset$ respectively, and therefore $u_{j,i} = 0$ or $y_{i,k} = 0$ accordingly. Finally, the simple structure of equations (2)–(3) allows for the interconnection between all the possible physical agents and therefore implement any type of network connection topology. They represent the switching of flows due to traffic signals or link saturations. The network thus defined becomes the plant of the system in the context of control theory. Thus, a control supervisor can be added according to the hybrid scheme shown in Figure 3, where the Petri Nets set the functions $\Gamma_{i,j}$ related to a certain traffic light cycle, and the plant provide the signals a_i of the saturation levels to be used by the supervisor. Traffic cycles are defined according to the type of intersections in the network. In the next two

Fig. 4. Example of a real intersection.

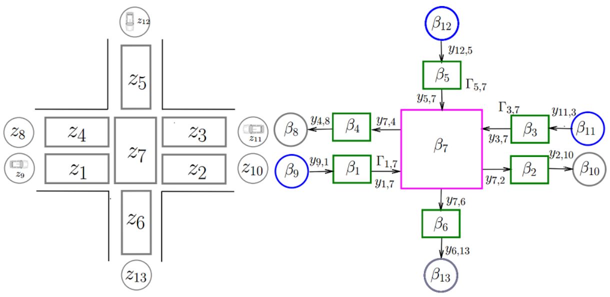

Fig. 5. Traffic network for the simulation example.

sections, an example is presented to show the performance of the approach.

V. Modeling of a single street intersection

The modular model of vehicle flow obtained in the previous section, is applied to a real intersection in Montevideo City composed by a simple four-road intersection, with one bidirectional and one unidirectional approach, as shown in Figure 4. The intersection is fed by three vehicle sources and discharged by three sinks. Using the formulation described in section III the four-road intersection is redrawn as shown in Figure 5. Thus, $n = 6$, $m = 1$, $k = 3$, $s = 3$ and consequently, reordering the subscripts, $\beta_{st} = \{\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6\}$, $\beta_{cr} = \{\beta_7\}$, $\beta_{so} = \{\beta_9, \beta_{11}, \beta_{12}\}$ and $\beta_{si} = \{\beta_8, \beta_{10}, \beta_{13}\}$. Since β_7 has a traffic light, a corresponding Petri Net PN_7 is set. Note that, by the interconnection of the segments shown by the arrows in Figure 5, the input and output adjacency subsets for each segment are given by

$$
\begin{aligned}
&N_1^+ = \{\beta_9\}, && N_1^- = \{\beta_7\}\\
&N_2^+ = \{\beta_7\}, && N_2^- = \{\beta_{10}\}\\
&N_3^+ = \{\beta_{11}\}, && N_3^- = \{\beta_7\}\\
&N_4^+ = \{\beta_7\}, && N_4^- = \{\beta_8\}\\
&N_5^+ = \{\beta_{12}\}, && N_5^- = \{\beta_7\}\\
&N_6^+ = \{\beta_7\}, && N_6^- = \{\beta_{13}\}\\
&N_7^+ = \{\beta_1, \beta_3, \beta_5\}, && N_7^- = \{\beta_2, \beta_4, \beta_6\}\\
&N_8^+ = \{\beta_4\}, && N_8^- = \emptyset\\
&N_9^+ = \emptyset, && N_9^- = \{\beta_1\}\\
&N_{10}^+ = \{\beta_2\}, && N_{10}^- = \emptyset\\
&N_{11}^+ = \emptyset, && N_{11}^- = \{\beta_3\}\\
&N_{12}^+ = \emptyset, && N_{12}^- = \{\beta_5\}\\
&N_{13}^+ = \{\beta_6\}, && N_{13}^- = \emptyset
\end{aligned}
\tag{8}
$$

Therefore, applying the equations (2)–(3) to each agent β_i, it becomes

$$\beta_1 : \begin{cases} \dot{z}_1 = u_{9,1} - a_7(z_7)\Gamma_{1,7}\alpha_1(z_1)z_1 \\ y_{1,7} = a_7(z_7)\Gamma_{1,7}\alpha_1(z_1)z_1 \end{cases} \tag{9}$$

$$\beta_2 : \begin{cases} \dot{z}_2 = u_{7,2} - a_{10}(z_{10})\Gamma_{2,10}\alpha_2(z_2)z_2 \\ y_{2,10} = a_{10}(z_{10})\Gamma_{2,10}\alpha_2(z_2)z_2 \end{cases} \tag{10}$$

$$\beta_3 : \begin{cases} \dot{z}_3 = u_{11,3} - a_7(z_7)\Gamma_{3,7}\alpha_3(z_3)z_3 \\ y_{3,7} = a_7(z_7)\Gamma_{3,7}\alpha_3(z_3)z_3 \end{cases} \tag{11}$$

$$\beta_4 : \begin{cases} \dot{z}_4 = u_{7,4} - a_8(z_8)\Gamma_{4,8}\alpha_4(z_4)z_4 \\ y_{4,8} = a_8(z_8)\Gamma_{4,8}\alpha_4(z_4)z_4 \end{cases} \tag{12}$$

$$\beta_5 : \begin{cases} \dot{z}_5 = u_{12,5} - a_7(z_7)\Gamma_{5,7}\alpha_5(z_5)z_5 \\ y_{5,7} = a_7(z_7)\Gamma_{5,7}\alpha_5(z_5)z_5 \end{cases} \tag{13}$$

$$\beta_6 : \begin{cases} \dot{z}_6 = u_{7,6} - a_{13}(z_{13})\Gamma_{6,13}\alpha_6(z_6)z_6 \\ y_{6,13} = a_{13}(z_{13})\Gamma_{6,13}\alpha_6(z_6)z_6 \end{cases} \tag{14}$$

$$\beta_7 : \begin{cases} \dot{z}_7 = u_{5,7} + u_{3,7} + u_{1,7} - \alpha_7(z_7)z_7[a_4(z_4)\Gamma_{7,4} \\ \quad +a_2(z_2)\Gamma_{7,2} + a_6(z_6)\Gamma_{7,6}] \\ y_{7,4} = a_4(z_4)\Gamma_{7,4}\alpha_7(z_7)z_7 \\ y_{7,2} = a_2(z_2)\Gamma_{7,2}\alpha_7(z_7)z_7 \\ y_{7,6} = a_6(z_6)\Gamma_{7,6}\alpha_7(z_7)z_7 \end{cases} \tag{15}$$

$$\beta_8 : \{\ \dot{z}_8 = a_8(z_8)\Gamma_{4,8}\alpha_4(z_4)z_4 \tag{16}$$

$$\beta_9 : \begin{cases} \dot{z}_9 = -a_1(z_1)\Gamma_{9,1}\alpha_9(z_9)z_9 \\ y_{9,1} = a_1(z_1)\Gamma_{9,1}\alpha_9(z_9)z_9 \end{cases} \tag{17}$$

$$\beta_{10} : \{\ \dot{z}_{10} = a_{10}(z_{10})\Gamma_{2,10}\alpha_2(z_2)z_2 \tag{18}$$

$$\beta_{11} : \begin{cases} \dot{z}_{11} = -a_3(z_3)\Gamma_{11,3}\alpha_{11}(z_{11})z_{11} \\ y_{11,3} = a_3(z_3)\Gamma_{11,3}\alpha_{11}(z_{11})z_{11} \end{cases} \tag{19}$$

$$\beta_{12} : \begin{cases} \dot{z}_{12} = -a_5(z_5)\Gamma_{12,5}\alpha_{12}(z_{12})z_{12} \\ y_{12,5} = a_5(z_5)\Gamma_{12,5}\alpha_{12}(z_{12})z_{12} \end{cases} \tag{20}$$

$$\beta_{13} : \{\ \dot{z}_{13} = a_{13}(z_{13})\Gamma_{6,13}\alpha_6(z_6)z_6 \tag{21}$$

Because of the interconnection between segments shown in Figure 5, some outputs become inputs. In this case,

$$
\begin{aligned}
y_{9,1} &= u_{9,1}, & y_{1,7} &= u_{1,7}, & y_{7,2} &= u_{7,2},\\
y_{11,3} &= u_{11,3}, & y_{3,7} &= u_{3,7}, & y_{7,4} &= u_{7,4},\\
y_{2,10} &= u_{2,10}, & y_{4,8} &= u_{4,8}, & y_{7,6} &= u_{7,6},\\
y_{12,5} &= u_{12,5}, & y_{5,7} &= u_{5,7}, & y_{6,13} &= u_{6,13}.
\end{aligned}
$$

Note that the activation functions $\Gamma_{9,1}, \Gamma_{7,2}, \Gamma_{2,10}, \Gamma_{11,3}, \Gamma_{7,4}, \Gamma_{4,8}, \Gamma_{12,5}, \Gamma_{7,6}$ and $\Gamma_{6,13}$ are equal to one, since there are no traffic lights. Similarly, the saturation functions $a_8 = a_{10} = a_{13} = 1$, at all time, since the maximal value $z_8^{\max} = z_{10}^{\max} = z_{13}^{\max} = \infty$.

The timed Petri Net PN_7 that implements a standard traffic light cycle for agent β_7 is shown in Figure 6. The places $\{p_1, p_2, p_3\}$ represent the green, yellow and red lights respectively, of the signal that controls the car flow from agent β_1 to the intersection β_7. Places $\{p_4, p_5, p_6\}$ represent the traffic signal that controls the vehicle flow from the street agent β_3 to β_7 while places $\{p_7, p_8, p_9\}$ control the vehicle flow from the street agent β_5 to β_7. The initial marking of the PN is $M_0 = [1, 0, 0, 1, 0, 0, 0, 0, 1]$ which represents that the places p_1 and p_4 have one token, so the green light is

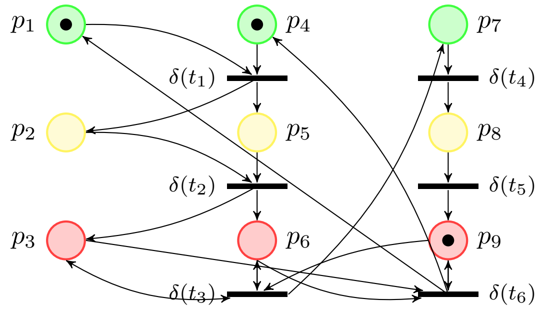

Fig. 6. Petri Net for traffic signal control of intersection agent β_7.

on for the signals that control the flow from β_1 and β_3 to β_7, thus $\Gamma_{1,7} = \Gamma_{3,7} = 1$. In a similar way p_9 has one token and this represents that the red light is on for the signal that controls the flow from β_5 to β_7 and $\Gamma_{5,7} = 0$. In the initial marking, the transition t_1 is enabled since there are tokens in p_1 and p_4. Thus, the tokens in p_1 and p_4 can be moved to p_2 and p_5 after a delay time $\delta(t_1)$, changing the traffic light from green to yellow. Afterwards, t_2 will be enabled, the tokens will move to p_3 and p_6 and all traffic lights will be in red after a time $\delta(t_2)$ making all flows to stop. In this state, transitions t_3 and t_6 are enabled, and after times $\delta(t_3)$ and $\delta(t_6)$ respectively, will cause tokens in p_3 and p_6 to remain there while token from p_9 will move to p_7. Then, $\Gamma_{5,7} = 1$ and the flow from β_5 to β_7 will be allowed. Following the same mechanics, transitions t_4 and t_5 will fire sequentially and the marking of the PN will move back to M_0. This cycle is repeated in the PN generating a fix-time control for the traffic network.

VI. Numerical simulation

The simulation was developed in Matlab, as mentioned before. The input data for the model belong to a real intersection at Montevideo city, where the sources $\beta_9, \beta_{11}, \beta_{12} \in \beta_{so}$ provide $1000, 1362$ and 200 vehicles per hour respectively. The segments of streets $\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6 \in \beta_{st}$ and $\beta_7 \in \beta_{cr}$ are initially empty. The maximum occupancy of cars of each segment of street is defined as: $z_1^{\max} = 157, z_2^{\max} = 66, z_3^{\max} = 66, z_4^{\max} = 157, z_5^{\max} = 25, z_6^{\max} = 29$ and $z_7^{\max} = 30$, in the case of sinks $z_9^{\max} = 1100, z_{11}^{\max} = 1400, z_{12}^{\max} = 300$. Figure 6 shows the transitions in PN_7 which have a time delay to be fired given by $\delta(t_1) = 20$, $\delta(t_4) = 40$, $\delta(t_2) = \delta(t_5) = 5$ and $\delta(t_3) = \delta(t_6) = 5$. According to equation (6) the definition of $\Gamma_{1,7} = C_{1,7}M_7$ with $C_{1,7} = [1, 0, 0, 0, 0, 0, 0, 0, 0]$. Similarly, $\Gamma_{3,7} = C_{3,7}M_7$ with $C_{3,7} = [1, 0, 0, 0, 0, 0, 0, 0, 0]$, $\Gamma_{5,7} = C_{5,7}M_7$ with $C_{5,7} = [0, 0, 0, 0, 0, 0, 0, 0, 1]$. Since the output flow from β_7 to β_2 depends on the input flows according to the turning ratios, $\Gamma_{7,2} = 0.94\Gamma_{1,7} + 0.04\Gamma_{3,7} + 0.0\Gamma_{5,7}$, $\Gamma_{7,4} = 0.0\Gamma_{1,7} + 1.0\Gamma_{3,7} + 0.0\Gamma_{5,7}$, and $\Gamma_{7,6} = 0.35\Gamma_{1,7} + 0.35\Gamma_{3,7} + 0.3\Gamma_{5,7}$. Therefore, $C_{7,2} = [0.94, 0, 0, 0.06, 0, 0, 0, 0, 0]$, $C_{7,4} = [0, 0, 0, 1, 0, 0, 0, 0, 0]$, and $C_{7,6} = [0.35, 0, 0, 0.35, 0, 0, 0.3, 0, 0]$. The simulation results are shown in Figures 7 to 9. Figure 7 displays the

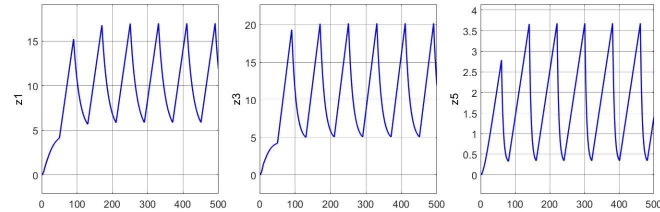

Fig. 7. Occupancy simulation response of agents $\beta_1, \beta_3, \beta_5 \in \beta_{st}$

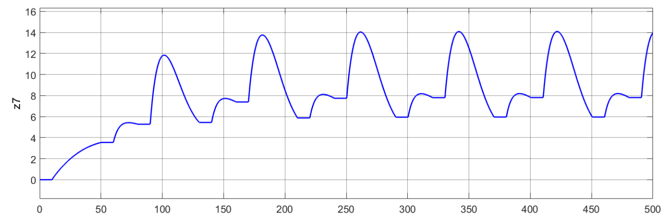

Fig. 8. Occupancy of agent $\beta_7 \in \beta_{cr}$

segments of street and $\beta_1, \beta_3, \beta_5 \in \beta_{st}$. Figure 8, shows the state of the intersection agent $\beta_7 \in \beta_{cr}$. Figure 9 shows the occupancy of $\beta_2, \beta_4, \beta_6 \in \beta_{st}$. Figure 10 shows the simulation of the traffic signal that controls the flow from β_1 and β_3 to β_7 and the traffic signal for the path from β_5 to β_7. Also observe, that all segments β_i do not saturate due to the traffic lights sequence in combination with the $\alpha_{i,k}$. Once the results shown in the previous graphs were obtained, a simulation of the same network shown in Figure 4 was carried out in a traffic simulation software package, TSIS-CORSIM, using the same data from the previous simulation. Figure 11 shows the comparison of $\beta_1, \beta_3, \beta_5 \in \beta_{st}$ obtained from the TSIS-CORSIM simulator with respect to our model, the blue color graph represents the TSIS-CORSIM data and

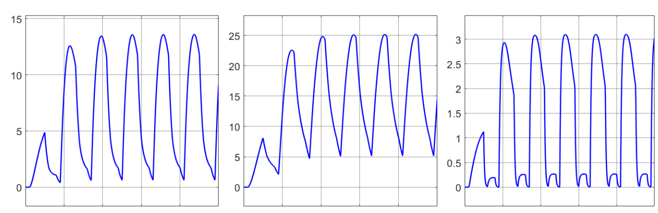

Fig. 9. Occupancy simulation response of agents $\beta_2, \beta_4, \beta_6 \in \beta_{st}$

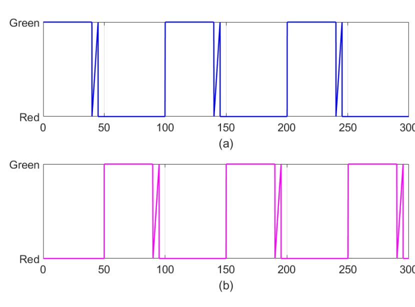

Fig. 10. Simulation of traffic lights for Γ_{17},Γ_{37} (a) and Γ_{57} (b).

the red one, our simulator. In the same way Figure 12 shows the comparison of $\beta_2, \beta_4, \beta_6 \in \beta_{st}$ for both simulators.

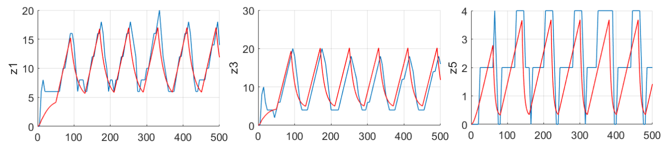

Fig. 11. Occupation in agents $\beta_1, \beta_3, \beta_5 \in \beta_{st}$ for both simulators.

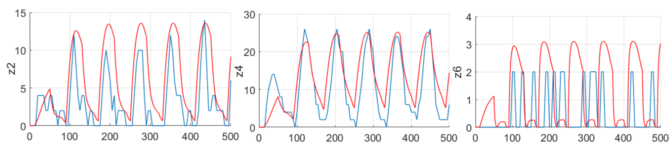

Fig. 12. Occupation in agents $\beta_2, \beta_4, \beta_6 \in \beta_{st}$ for both simulators.

VII. Conclusions and Future Work

This work introduces a macroscopic model of urban traffic networks using a hybrid architecture composed of a multi agent system and timed PNs. It features a flow model that varies not only with street occupancy but also with downstream link state to model congestion issues. The Timed PNs that manage the traffic lights allow for the introduction of complex supervisory control algorithms to optimize network flows. The architecture is designed to be modular and scalable to help realize larger networks with different topologies in a simple way. It differs from a hybrid automata architecture in the use of PNs with a unified dynamical model. This architecture has similarities with MLD system architecture, but defines the logical variables through timed PNs. Simulation results with real data from an intersection in the city of Montevideo are comparable to a known simulator such as CORSIM. Future work includes the simulation of a complex network and further tests against CORSIM with real data to optimize flows and reduce congestion. Future research also aims to design control algorithms that handle usual traffic issues such as congestion, peak hour traffic and generation of green waves.

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