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# A multiagent systems with Petri Net approach for simulation of urban traffic networks

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ABSTRACT

This paper presents a novel model framework for complex urban traffic systems based on the interconnection of a dynamical multi-agent system in a macroscopic level. The agents describe all the types of street segments, intersections, sources and sinks of cars, modelling the behavior of the flow of vehicles through them as simple differential equations. These agents include the phenomena of changes in the flow rate due to congestions, traffic signals and the density of the vehicles. Traffic signal changes are obtained by the evolution of Petri Nets, in order to represent a more real behavior. Therefore, a complex network can be constructed by the interconnection of the agents, in continuous time, and the Petri Nets, in a discrete-event behavior, becoming a hybrid and scalable system. In order to analyze the performance of the approach, a real set of streets and intersections in Montevideo City is studied. Also, the approach is compared with a simulation realized in the software TSIS-CORSIM, which contains real data of density of vehicles. The multi-agent system achieves comparable results, taking into account the differences in the level of details respect to TSIS-CORSIM. Thus, the results can represent the most important issues of vehicular traffic with less computational resources.

## 1. Introduction

The main focus of a Multi-Agent Dynamical Systems (MAS) is to represent the behavior of complex systems as the concurrent behavior of its dynamical elements, called *agents* (Axtell, 2016). The main challenge is the definition of each agent and its information that must be shared with other agents according to a communication topology (Teng, Zhang, Luo, & Shan, 2019). The dynamics of an agent can be modeled using differential equations, probability, discrete-event evolution, among others. The inter-agent communication depends on the physical features of the systems, for instance, the proximity of agents, the wireless communication coverage, etc. (Wang, Cao, Wang, & Alsaadi, 2019; Yang, Zhang, He, Han, & Peng, 2020). The MAS formalism has been useful to represent the coordination motion of mobile robots, large economic systems, project task planning, determination of optimal routes (Wang & Zlatanova, 2016), social networks, etc. An agent-based approach applied to traffic systems is proposed in (Liu, Baldi, Yu, & Chen, 2020) by partitioning a complex system into agents and the design of heuristic algorithm for solving the switch-based adaptive dynamic programming (ADP) in a distributed way. This paper intends to apply the concepts of MAS to the context of Urban Traffic Systems (UTS).

The UTS in large cities has become an important issue due to the growth in the world's population and vehicles, generating different problems for the health and mobility of people. Some consequences are the air pollution (Yanga, Mab, & Sunb, 2018), smog and haze (Xie et al., 2019), respiratory problems (Sharma, Kharol, & Badarinath, 2010; Silva, Ball, Huynh, & Reichman, 2016) as well as economic and social problems, among others as mentioned in (Grote, Williams, Preston, & Kemp, 2016; Jia, Yan, & Shen, 2018; Yolton et al., 2019). These problems have been addressed using different approaches to try to mitigate

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the effects mentioned above, for example in (Chew & Wu, 2016) the main sources of noise are identified, as well as the spread of it (Nega, Smith, Bethune, & Fu, 2012).

The dimension of a UTS becomes complex in some places, for example, according to (I. N. de Estadística y Geografía, 2017) for the case of Mexico Metropolitan Area, around 20.8 millions of inhabitants live within an accelerated urban growth and 34.6 million trips are made per week to the downtown area.

The study of UTS in order to optimize the flow of vehicles can be decomposed in two main steps. Firstly, the complex dynamics of UTS must be modeled representing the common phenomena in the network, for instance, the congestions, the speed limits, the average flow according to the time of day, the possible number of vehicles in streets and intersections, etc. (Goñi-Ros et al., 2016; Klawtanong & Limkumnerd, 2019; Kohan & Ale, 2020; Zhang, Xue, Zhang, Fan, & di He, 2019). In a second step, some control algorithms must be designed in order to regulate adequately the flow of vehicles using the traffic lights, planning of trips, searching of alternative flows due to car crashes, accidents, road obstructions, etc. (Nagatani, 2018; Nagatani & Tobita, 2012; Otero, Galetti, & Mizrahi, 2018). For example, in (Liu, Yu, Baldi, Cao, & Huang, 2020) a switching-based control formulation is proposed for multi-intersection and multi-phase traffic light systems in a macroscopic level, taking into consideration structural and nonlinear characteristics. It is clear that an adequate UTS control system can be achieved if there exists a sufficiently good model of the network.

The literature about UTS modelling is large and diverse with different levels of detail and using distinct mathematical tools. A very complete review is found in (Chen & Cheng, 2010). A common classification of UTS models is related to microscopic and macroscopic level of detail (Hoogendoorn & Bovy, 2001; van Wageningen-Kessels, van Lint, Vuik, & Hoogendoorn, 2015a). Some works consider a mesoscopic level, as a combination of the micro and macroscopical representation.

The microscopical viewpoint represents the cars as particles, with local information about their speed, acceleration, type and size of the vehicle, etc. (Bellemans, Schutter, & Moor, 2002). Some heuristic rules of the behavior of the drivers are designed to preserve an average speed, the distance respect to other cars, etc. (Borsche, 2019; Yu, Kamel, Gong, & Li, 2014). Some software applications have been developed for the microscopic level (Barceló, 2010; Hossain & McDonald, 1998; Maecki & Iwan, 2019), for instance, TSIS-CORSIM (Traffic Software Integrated System, CORridor SIMulation), AIMSUN and SUMO (Simulation of Urban Mobility), among others, where it is possible to visualize the motion of all the cars moving in the network. However, the computational cost is very high when it is used in large UTS networks. Also, some formal properties and features cannot be obtained because a global model of the network does not exist, and the dynamics depends on the initial conditions and some random features.

The models of UTS in a microscopical viewpoint have also been addressed using a discrete-event representation. A common formalism for UTS is the Petri Nets (PN) (dos Santos Soares & Vrancken, 2012; Ng, Reaz, & Ali, 2013; Wang, Li, & Cai, 2012). Some relevant works are (Azzumar, Halim, & Harjono, 2013a; Basile, Chiacchio, & Teta, 2012; Chen, Wang, Jiang, & Wen, 2014)), where a timed and colored PN describes the urban network and each position in the streets and intersections is represented by a token in a *place* node. This level of detail decomposed the physical space in position cells occupied or not by vehicles. A similar focus is given in (DiFebbraro, Giglio, & Sacco, 2016), where a supervisory control for PN is designed to avoid deadlock effects in crosses and the optimization of times by the minimization of a function of the PN marking. In (Qi, Zhou, & Luan, 2015) is addressed the coordination of traffic lights by the occurrence of congestion and incidents in crosses, where the dynamics of the flow of vehicles is obtained by simple sensors. Note that the PN increases for large real networks. Therefore, (Qi et al., 2015) considers simple intersections only with difficulties to be scalable. A similar focus is the called *cell transmission model*, where the street segments are decomposed in cells, analyzing the input and output of flows in simple discrete-time equations (Xie, Xu, Härri, & Chen, 2013).

On the other hand, in the macroscopic level, the vehicular traffic is considered as a flow of vehicles. Thus, the main variable is the density of flow (average of vehicles in a street segment). It allows to create state space models in continuous and discrete time (Kawasaki, Hara, & Kuwahara, 2017). A similar focus is the mesoscopic level, where the flow of cars can be analyzed by lots or groups of cars, for example in (Yuan, Duret, & van Lint, 2015).

In the literature of UTS modelling in a macroscopical level, the flow of vehicles becomes a set of equations. For instance, in (Wang, Dai, Xiaoming, & Zheng-xi, 2013) a set of streets are modeled as a discrete-time linear state-space system. The effects of the traffic lights are incorporated as simple constant parameters related to the average of the distribution of the flow. Different signal control strategies have been developed, for example in (Aboudolas, Papageorgiou, Kouvelas, & Kosmatopoulos, 2010), a signal control methodology to a large-scale urban traffic network is proposed, based on computationally feasible quadratic-programming control. In (Wey, 2000) a model is proposed that contemplates traffic signals, seeking to optimize them in a vehicular network. A distinct focus is given in (Kong, Tu, & Liu, 2012) where the streets and intersections are simple coordinates, containing a simple sum of flows. The focus of consensus in a discrete-time representation is given in (Zhonghe, Chi, & Li, 2015), where a state-based feedback controller is designed for the balance of the flow of vehicles in the streets, considering the control inputs as green time percentage respect to the traffic light cycle. A traffic control strategy is proposed in (Aboudolas, Papageorgiou, & Kosmatopoulos, 2009), combining store-and-forward traffic flow modelling, mathematical optimization and optimal control through a methodology of an open-loop constrained quadratic optimal control problem. In (Baldi, Michailidis, Kosmatopoulos, Papamichail, & Papageorgiou, 2017), another adaptive traffic responsive strategy to control traffic signals was developed, an algorithm maximizes a performance index composed of mean speed and total travel distance. Finally, in (Danilevičius & Bogdevičius, 2017) analyses in discrete-time the variations of the velocities and concentrations of segments of roads, with distinct traffic lights switching periods.

For the case of hybrid approaches, some works as (Tolba, Lefebvre, Thomas, & Moudni, 2005) use a combination of continuous and timed PNs along with an adaptive spatial discretization equation, become a model for traffic flow in a macroscopic level. Hybrid agent-based modelling are proposed in (Manley, Cheng, Penn, & Emmonds, 2014) to represent the vehicular flow in a complex networks. Hybrid system approaches such as hybrid automata (Jiang, Chen, Zhang, & Li, 2009) or mixed logical dynamical (MLD) models have also been applied (Pahnabi, Karimpour, & Mobara, 2016). The focus of (Zhou, De Schutter, Lin, & Xi, 2017) is to decompose the UTS in two hierarchical levels, where the upper level considers subnets of UTS, whereas the lower level applies a distributed Model Predictive Control to regulate the flow with the green-time percentages. Delay lines representing the flow in intersections combined with PN as traffic lights is proposed in (Renato Vázquez, Sutarto, Boel, & Silva, 2010). A continuous and hybrid PN is used in (Azzumar, Halim, & Harjono, 2013b), where a continuous PN model is adjusted by real data and the flow of vehicles is represented by a continuous marking vector.

The main objective of this work is to propose a hybrid model for UTS using a MAS framework to describe the topology and dynamics of the traffic network with a set of different Petri Nets to control the traffic signals that manage vehicles flows. The main contributions of this work can be summarized as follows:

- Each element of the traffic system, for example street segments, intersections, vehicle sources and sinks, is defined as a dynamical agent using simple and modular differential equations to describe the vehicle flows.

- The dynamics of the agents take into account the effects of the traffic lights, the street and intersection saturation levels, the relation between vehicle density in the streets and flows, propagation times and the congestion phenomena in the network.
- The complexity of the network is generated by the interconnection of the agents described by the MAS communication topology.
- PNs are used to model the traffic light behavior for each type of intersection to optimize the vehicle flows in the traffic network.
- The simulator of this hybrid approach is developed using Matlab, in order to show the scalability of the model, establishing the core of a novel vehicular traffic simulator.
- Validate the approach using a CORSIM microscopic-based simulation with data from a real UTS network.

The last item is inspired by works like (Xie et al., 2013), where a macroscopical approaches from a real case of study is validated with microscopical-based software like SUMO and CORSIM in (Kong et al., 2012; Zhou et al., 2017).

The paper combines strategically continuous flow equations with Petri Nets to obtain a compact model for the traffic lights and good macroscopical behavior of flows, respectively. The approach becomes original by the next features:

- The traffic lights are modeled using a timed discrete-event representation which is the most faithful description of the behavior of a real traffic light. It is common that the works related to continuous equations of traffic flow, the effects of the traffic lights are reduced to control inputs related to the flow distribution in the streets and intersections.
- The flow equations are designed to be modular for easier interconnection. The approach avoids to represent these flows by Petri Nets. It is common that when the Petri Nets are used as traffic ligths, the flow of cars are modeled as Petri Nets too, in order to avoid a hybrid modelling. However, the resources grows exponentially fast when the network increases. Thus, most of the papers based only in Petri Nets analyze small networks composed by one intersection or a few number of streets.
- The hybrid modelling is designed to be flexible with respect to a multi-agent analysis or supervisory control of the traffic lights. Some structural network properties like Laplacian matrices, adjacency subsets, graph switching, among others can be identified in further works in order to achieve a control for the system. It is different of other works where the models are unique and non scalable to address large networks.

The article is organized as follows, fundamental concepts of PN are given in Section 2. Section 3 introduces the problem formulation while the modular equations of traffic flow are developed in Section 4. The main application to a simulation of a network in Montevideo city is presented in Section 5. Finally, conclusions and future work are outlined in Section 6.

## 2. Petri Net definitions

Petri Nets (PN), developed by C.A Petri, are related to automatons in the sense that they represent transitions of discrete event systems (DES), through certain rules. Petri Nets include conditions under which, events can be enabled, allowing complex control schemes to be generated. PN have been studied extensively in urban traffic control systems (Chen & Cheng, 2010), since they have several techniques for their analysis, which represents an active research topic.

Petri Nets consist of directed graphs, with two types of nodes (places and transitions). Actions or events are represented in places and events by transitions. An extension of a PN called Timed Petri-Net *TPN* is defined by the six-tuple

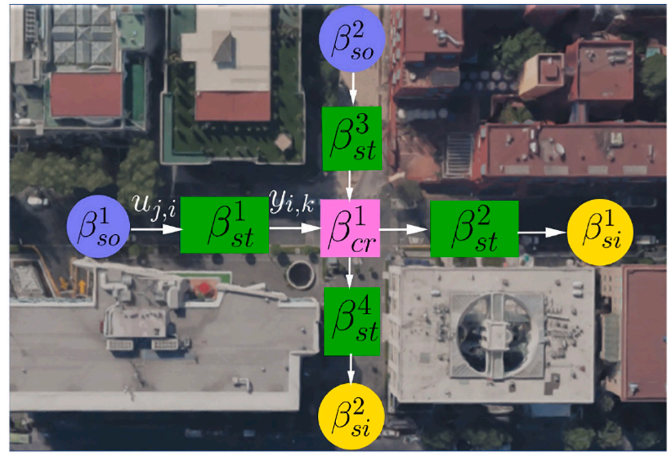


**Fig. 1.** Example of a simple intersection.

$$TPN = (P, T, A, w, M_0, \delta), \tag{1}$$

where $P = \{p_1, p_2, \ldots, p_n\}$ are called *Places*, a finite set of $n$ nodes in a graph, $T = \{t_1, t_2, \ldots, t_m\}$ represent a finite set of $m$ nodes called *Transitions*. $P$ and $T$ are connected by *arcs* $A \subseteq (P \times T) \cup (T \times P)$ with positive integer weights $w : A \to \mathbb{Z}^+$. The initial marking $M_0$, of the PN is defined by the number of tokens contained in each place $p \in P$. The number of tokens inside each place at time $k$ allows to define a $n$-th vector $M_k(p) : P \to \mathbb{Z}^+$. A transition rule, establishing the $k$-th state or marking of the PN is defined as follows: transition $t_i \in T$ is said to be enabled in a PN with finite capacity if: $M_k(p_j) \geq w(p_j, t_i), \forall p_j \mid w(p_j, t_i) \in w$, with $j = 1, \ldots, n$ and $i = 1, \ldots, m$. If $t_i$ is enabled the next marking is defined as $M_{k+1}(p_j) = M_k(p_j) - w(p_j, t_i) + w(t_i, p_j)$.

The principal characteristic of a *TPN* is the assignment of a positive number $\tau_i$ to each transition $t_i$ in the net by the function $\delta : T \to \mathbb{R}^+$. The term $\tau_i = \delta(t_i)$ defines the time delay until a transition $t_i$ is enabled and the tokens are moved to their next place $M_{k+1}(p), \forall p \in P$.

## 3. Problem definition

This work proposes to describe an urban traffic network as a multi-agent system consisting in the interconnection of the different elements in the network.

In this way let $\beta = \beta_{st} \cup \beta_{cr} \cup \beta_{so} \cup \beta_{si}$ be the set of all agents in the network, where $\beta_{st} = \{\beta_{st}^1, \ldots, \beta_{st}^n\}$ is the set of $n$ street agents, $\beta_{cr} = \{\beta_{cr}^1, \ldots, \beta_{cr}^m\}$ is the set of $m$ intersection agents, $\beta_{so} = \{\beta_{so}^1, \ldots, \beta_{so}^k\}$ is the set of vehicle sources and $\beta_{si} = \{\beta_{si}^1, \beta_{si}^2, \ldots, \beta_{si}^s\}$ is the set of car sinks. Vehicles enter the network from different sources, which include physical spaces like garage parking, building parking or new car agencies. The outflow of cars from the network is concentrated on the sink agents. An example of a real vehicular intersection is shown in Fig. 1. The definition of sources and sinks, permit to establish the total number of cars as a known quantity, regardless of the places where cars move, that is, the number of cars in the network is maintained. Also, it helps to construct autonomous differential equations, as it will be explained below.

The interconnections between the agents can be defined as a directed graph $\mathbb{G}$. Let $N_i^+ \subset \beta$ be the in-adjacency set of $\beta_i$, i.e. all the physical spaces that provide cars to $\beta_i \in \beta$. Similarly, consider $N_i^- \subset \beta$ as the out-adjacency set of $\beta_i$, i.e. the physical spaces that receive cars from $\beta_i \in \beta$. In this way, all possible vehicle exchanges between streets, intersections, sources and sinks are considered.

The number of cars in each agent $\beta_i \in \beta$ is defined as $z_i \in \mathbb{R}$. Also, let the function $\alpha_i(z_i) \in \mathbb{R}, \forall \beta_i \in \beta$ represents the inflow and outflow rates depending on the density of cars in the agent to be specified in Section 4.

A set of timed Petri Nets $\text{PN}_i$, $\forall \beta_{cr}^i \in \beta_{cr}$ is defined to manage the traffic flow of vehicles in the signalized intersections of the network.

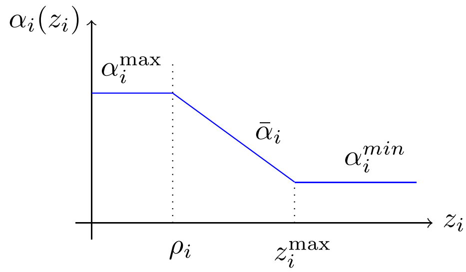


**Fig. 2.** Definition of $\alpha_i(z_i)$.

***Problem statement***: Define a hybrid model to describe the dynamic evolution of the occupation $z_i$ of the vehicles in a traffic network given by the agents $\beta_i \in \beta$ interconnected by the graph $\mathbb{G}$ and its associated adjacency sets $N_i^+$, $N_i^- \subset \beta$ and a supervisory control defined by the TPN $\text{PN}_i$ for each signalized intersection $\beta_{cr}^i$.

In this hybrid scheme, the closed-loop system between the plant (differential equations) and its supervisory control ($\text{PN}_i$) makes it possible to study the effects of the network topology and control on the vehicle density and distribution over the network, congestion, real cycles of the traffic lights, among others.

## 4. Modular equations of traffic flow

According to the previous section, a complex traffic network can be decomposed by the interconnection of the agents $\beta_i \in \beta$. Depending on the number of agents $\beta_i$, some typical intersections can be constructed, like *Y* type, *T* type, *Four Legs*, etc. and the mixing of all the possible intersections can build a complete city network.

In a macroscopical focus, as the main purpose of this paper mentioned in the Section 3, the dynamics of the agents must be represented by the evolution of the variables $z_i$, related to the density of vehicles in the spaces $\beta_i$. The dynamical equation of $z_i$ must include the effects of the congestions, the switching of the traffic lights, among other flow phenomena of the network. The definition of this modular differential equation of $z_i$ is one of the most important contributions of this work. The dynamics of the $z_i$ is defined by the next equation

$$\dot{z}_i = \sum_{\beta_j \in N_i^+} u_{j,i} - \sum_{\beta_k \in N_i^-} y_{i,k} \tag{2}$$

$$y_{i,k} = \left[\alpha_i(z_i) a_k(z_k) \Gamma_{i,k}\right] z_i, \forall \beta_k \in N_i^- \tag{3}$$

where $z_i$ is the density of vehicles in $\beta_i \in \beta$, $u_{ji}$ is the input flow to agent $\beta_i$ from $\beta_j \in N_i^+$ and $y_{i,\,k}$ is the output flow from $\beta_i$ to agent $\beta_k \in N_i^-$.

Note that the dynamics of $z_i$ given by Eq. (2) is based on the sum of the input and output flows to and from agent $\beta_i$. The nonlinear effects are contained in the output flow dynamics in Eq. (3). The output $y_{i,\,k}$ can be conceived as a discharge of $z_i$ with a parameter of discharge that depends on the multiplication of the terms $\alpha_i(z_i)$, $\Gamma_{ik}$ and $a_k(z_k)$. Note that the structure of Eqs. (2)–(3) is designed to be modular, where the output $y_{i,\,k}$ can become in the input $u_{k,\,m}$ of a next agent $\beta_m$.

The function $\alpha_i(z_i)$ in (3) is related to the change of the rate of discharge according to the density $z_i$. The main idea is to decrease gradually the discharge of vehicles as $z_i$ fills up. This effect is common in street segments, and it is common in the traffic modelling literature, as presented in (van Wageningen-Kessels, van Lint, Vuik, & Hoogendoorn, 2015b). The definition of $\alpha_i(z_i)$ in this paper is the linear function shown in the Fig. 2 and can be expressed as.

$$\alpha_i(z_i) = \begin{cases} \alpha_i^{\max}, & \text{if } z_i < \rho_i \\ \overline{\alpha}_i, & \text{if } \rho_i < z_i < z_i^{\max} \\ \alpha_i^{\min}, & \text{if } z_i = z_i^{\max} \end{cases} \tag{4}$$

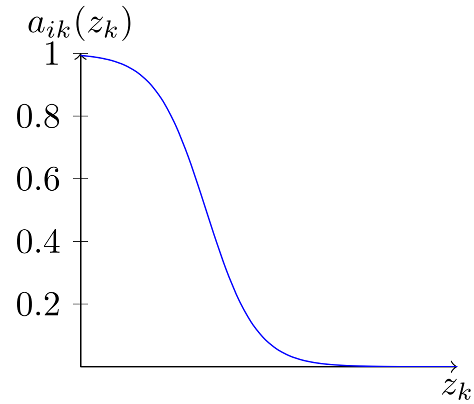


**Fig. 3.** Graph of the function $a_{ik}$.

where $z_i^{\max}$ is the maximum density of cars for $\beta_i$, at $\rho_i$ the function $\alpha_i$ begins to linearly decrease according to

$$\overline{\alpha}_i = \left(\frac{\alpha_i^{\min} - \alpha_i^{\max}}{z_i^{\max} - \rho_i}\right)(z_i - \rho_i) + \alpha_i^{\max} \tag{5}$$

to a minimum value $\alpha_i^{\min} < \alpha_i^{\max}$.

The area of the agent $\beta_i$ is based on the physical length of streets, number of lanes, average vehicle size, etc. and it defines the value of $\alpha_i^{\max}$. Note the rate of discharge flow decreases when the number of vehicles in the agent $\beta_i$ increases. There exist other similar functions in the literature, as presented in (van Wageningen-Kessels et al., 2015b).

The output flows (3) can be enabled or disabled by the activation functions $\Gamma_{i,\,k}$ and $a_k$. The function $\Gamma_{i,\,k}$ is related to a state vector of a Petri Net, if a traffic light exist, or remains in a value of 1, in other cases. Thus, $\Gamma_{i,\,k}$ is defined as

$$\Gamma_{i,k} = \begin{cases} C_{i,k} M_k, & \text{if } \beta_k \in \beta_{cr} \\ C_{i,k} M_i, & \text{if } \beta_i \in \beta_{cr} \\ 1, & \text{otherwise} \end{cases} \tag{6}$$

where $M_k \in \mathbb{R}^{n_k \times 1}$ is the actual marking vector of the PN of $n_k$ places representing the traffic light behaviors for intersection $\beta_k$, and $C_{i,\,k} \in \mathbb{R}^{1 \times n_k}$ is a vector with weighted parameters that can be used to specify the proportional flow in each output $y_{i,\,k}$. Thus, the function $\Gamma_{i,\,k}$, is related to a traffic light cycle if the output $y_{i,\,k}$ connects the segment $\beta_i$ to an intersection agent $\beta_k$ supervised by a traffic light, and it is obtained as a linear combination of the vector $M_k$. If there exists no traffic light for $\beta_k$ or $\beta_k \notin \beta_{cr}$, then the function $\Gamma_{i,\,k}$ is always activated, allowing a free flow of vehicles.

Finally, the functions $a_k(z_k)$ in Eq. (3) represent a feedback information to control the output flow of $\beta_i$ according to the saturation level of the downstream segment $\beta_k \in N_i^-$. The function selected is:

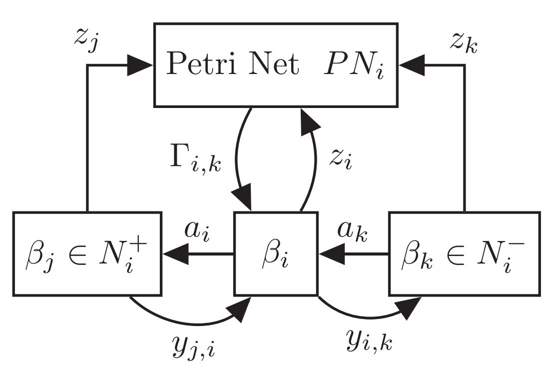


**Fig. 4.** Hybrid architecture proposed for a traffic network.

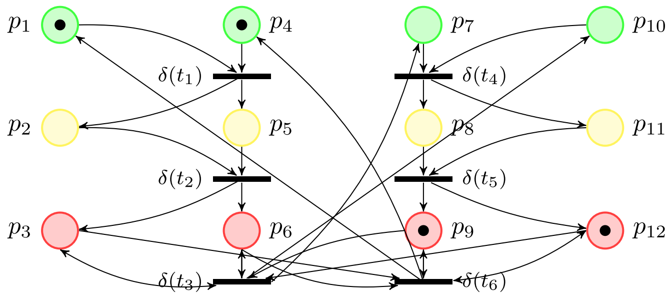


**Fig. 5.** Petri Net for traffic signal control of intersection agent $\beta_{17}$.

$$a_k(z_k) = \frac{1}{1 + exp\left(k_e\left(\frac{z_k}{z_k^{max}} - \xi\right)\right)}, \tag{7}$$

where $k_e, \xi \in \mathbb{R}$, is shown in Fig. 3. When $\beta_k$ is full, $a_k \approx 0$, and the output flow of the precedent agent $\beta_i$ is blocked. Eqs. (2)–(3) apply directly to streets and intersections, i.e. $\forall \beta_i \in \beta_{st}, \beta_{cr}$, but they can also be applied to sources $\beta_i \in \beta_{so}$ and sinks $\beta_i \in \beta_{si}$ where $N_i^+ = \varnothing$ and $N_i^- = \varnothing$ respectively, and therefore $u_{j,\,i} = 0$ or $y_{i,\,k} = 0$ accordingly.

As mentioned before, the model of traffic networks is a modular setup, where Eqs. (2)–(3) allows the interconnection between different agents and thus different topology forms of networks. The network thus defined becomes the plant of the system in the context of control theory. Thus, a control supervisor can be added according to the hybrid scheme shown in Fig. 4, where the Petri Nets set the functions $\Gamma_{i,\,j}$ related to a certain traffic light cycle, and the plant provide the signals $a_i$ of the saturation levels to be used by the supervisor. Traffic cycles are defined according to the type of intersections in the network. In the next two sections, an example is presented to show the performance of the approach.

The advantage of our approach using continuous differential equations to describe the macroscopical behavior of the flow of cars is the insight develop from physics to model dynamics associated with fluids. Also, the interactions between the different modular flows do not incur in any artificial rules due to the lack of physics generated from a sample-based description of a discrete-time representation. Thus, it helps in a better numerical simulation of the agents. A disadvantage of this continuous representation is the major computational load to execute the simulations. Nevertheless, the problematic issues related to sampling are not presented in the continuous model.

## 5. Model validation

In our previous works (Flores-Geronimo, Hernandez-Martinez, Ferreira-Vazquez, Flores-Godoy, & Fernandez-Anaya, 2018), a simple example of a using a simple unidirectional intersection of four branches was modeled, with two sources and two sinks. In this simulation it can be seen that the flow of cars is distributed in the network, as well as the sources decline and the sinks are increased, both, with a constant rate. On the other hand, a simple intersection with unidirectional and bi-directional flow was modeled in (Flores-Geronimo, Hernandez-Martinez, Ferreira-Vazquez, Flores-Godoy, & Fernandez-Anaya, 2019), with three sources and three sinks, it is highlighted that the street segments do not show saturation due to traffic signals sequence in combination with $\alpha_{ik}$.

### 5.1. Traffic network 1

In this paper, the approach is validated using the data from a real intersections located in Montevideo city. It consists of three signalized vehicular crossings, with bidirectional and unidirectional approach, as shown in Fig. 7. The intersection is fed by six vehicle sources $\beta_{20}$, $\beta_{22}$, $\beta_{24}$, $\beta_{26}$, $\beta_{28}$, $\beta_{30} \in \beta_{so}$, and discharged by six sinks $\beta_{21}$, $\beta_{23}$, $\beta_{25}$, $\beta_{27}$, $\beta_{29}$, $\beta_{31} \in \beta_{si}$, as seen in Fig. 8. Thus, the sources on *8 de Octubre Avenue* $\beta_{20}$ and $\beta_{26} \in \beta_{so}$ provide 1300, 1362 vehicles per hour respectively. The sources on *Garibaldi Avenue*, $\beta_{30}$ and $\beta_{22}$ provide 350 cars per hour each one. $\beta_{28}$ on *Estero Bellaco*, and $\beta_{24}$ on *Joaquín Secco Illa* $\in \beta_{so}$ provide 200 vehicles per hour each one. The segments of streets $\beta_1$ to $\beta_{16} \in \beta_{st}$ and $\beta_{17}, \beta_{18}, \beta_{19} \in \beta_{cr}$ are initially empty. The maximum occupancy of cars of each segment of street on *8 de Octubre Avenue* are defined as: $z_1^{max}$, $z_8^{max} = 47$, $z_2^{max}$, $z_7^{max} = 60$, $z_3^{max}$, $z_6^{max} = 45$, $z_4^{max}$, $z_5^{max} = 45$. On *Garibaldi Avenue* $z_9^{max}$ to $z_{12}^{max} = 42$, and $z_{13}^{max}$ to $z_{16}^{max} = 22$. Each intersection of the vehicular network, has associated traffic signals, an example of Petri Net associated to intersection in *8 de Octubre Avenue* and *Garibaldi Avenue* is

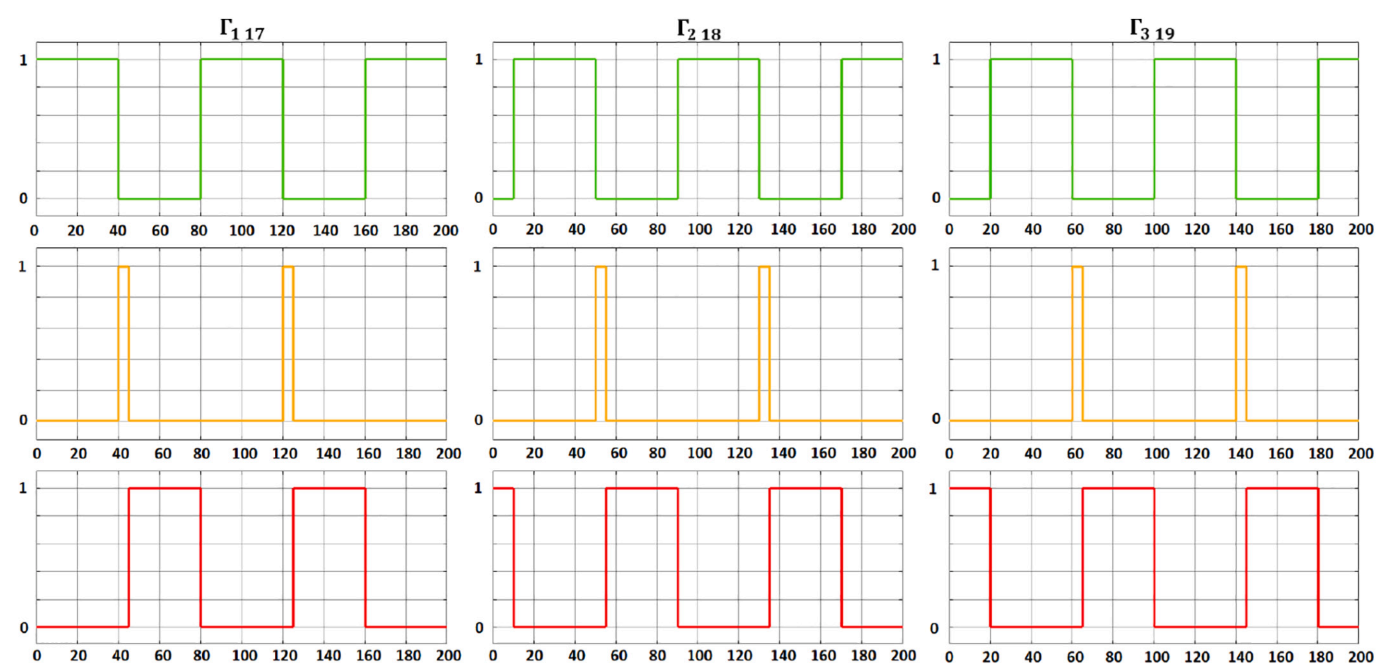


**Fig. 6.** Traffic signals of $\Gamma_{1,\,17}$, $\Gamma_{2,\,18}$ and $\Gamma_{3,\,19}$.

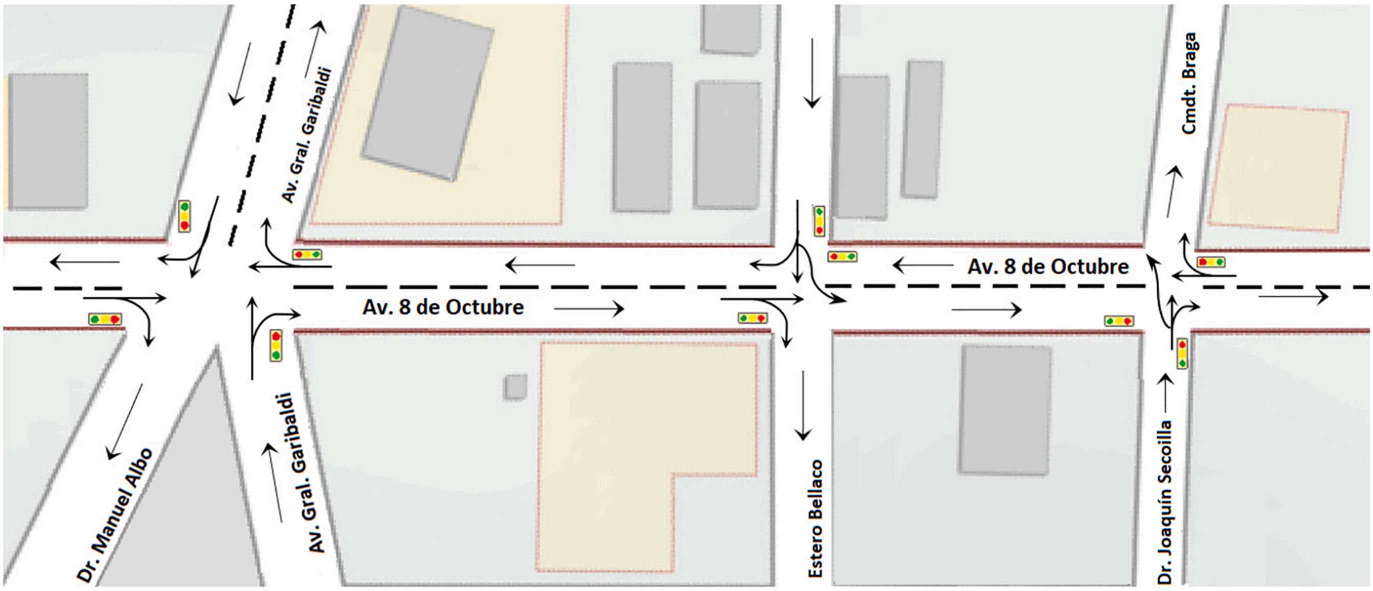


**Fig. 7.** Real vehicular network at Montevideo city.

shown in the Fig. 5. The transitions in each $PN_n$ of this network have a time delay to be fired given by $\delta(t_1) = 40$, $\delta(t_4) = 20$, $\delta(t_2) = \delta(t_5) = 5$ and $\delta(t_3) = \delta(t_6) = 5$.

Fig. 6 shows traffic signals $\Gamma_{1,\ 17}$, $\Gamma_{2,\ 18}$, $\Gamma_{3,\ 19}$ that enable the flow from agent $\beta_1$ to $\beta_4$ on *8 de Octubre Avenue*.

According to Eq. (6), to the crossing of *8 de Octubre Avenue* and *Garibaldi*, the respective $\Gamma_{ik}$ are defined as:

$\Gamma_{1,\ 17} = C_{1,\ 17}M_{17}$ with $C_{1,\ 17} = [1{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0]$
$\Gamma_{7,\ 17} = C_{7,\ 17}M_{17}$ with $C_{7,\ 17} = [1{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0]$
$\Gamma_{12,\ 17} = C_{12,\ 17}M_{17}$ with $C_{10,\ 17} = [0{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}1]$
$\Gamma_{10,\ 17} = C_{10,\ 17}M_{17}$ with $C_{10,\ 17} = [0{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}1]$

On *8 de Octubre Avenue* and *Estero Bellaco*

$\Gamma_{2,\ 18} = C_{2,\ 18}M_{18}$ with $C_{2,\ 18} = [1{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0]$
$\Gamma_{6,\ 18} = C_{6,\ 18}M_{18}$ with $C_{6,\ 18} = [1{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0]$
$\Gamma_{14,\ 18} = C_{14,\ 18}M_{18}$ with $C_{14,\ 18} = [0{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}1]$

On *8 de Octubre Avenue* and *Joaquín Secco Illa*

$\Gamma_{3,\ 19} = C_{3,\ 19}M_{19}$ with $C_{3,\ 19} = [1{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0]$
$\Gamma_{5,\ 19} = C_{5,\ 19}M_{19}$ with $C_{5,\ 19} = [1{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0]$
$\Gamma_{15,\ 19} = C_{15,\ 19}M_{19}$ with $C_{15,\ 19} = [0{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}0{,}1]$

The flow output at each intersection $\beta_{17}$ to $\beta_{19} \in \beta_{cr}$, depends on the input flows, in accordance with the following turning ratios, in the case of $\beta_{17} \in \beta_{cr}$, is defined as:

$$\Gamma_{17,2} = 0.94\Gamma_{1,17} + 0\Gamma_{7,17} + 0\Gamma_{12,17} + 0.06\Gamma_{10,17}$$

$$\Gamma_{17,8} = 0\Gamma_{1,17} + 0.94\Gamma_{7,17} + 0.06\Gamma_{12,17} + 0\Gamma_{10,17}$$

$$\Gamma_{17,11} = 0\Gamma_{1,17} + 0.06\Gamma_{7,17} + 0\Gamma_{12,17} + 0.94\Gamma_{10,17}$$

$$\Gamma_{17,9} = 0.06\Gamma_{1,17} + 0\Gamma_{7,17} + 0.94\Gamma_{12,17} + 0\Gamma_{10,17}$$

Therefore

$$C_{17,2} = [0.94, 0, 0, 0, 0, 0, 0, 0, 0, 0.06, 0, 0]$$

$$C_{17,8} = [0, 0, 0, 0.94, 0, 0, 0.06, 0, 0, 0, 0, 0]$$

$$C_{17,11} = [0, 0, 0, 0.06, 0, 0, 0, 0, 0, 0.94, 0, 0]$$

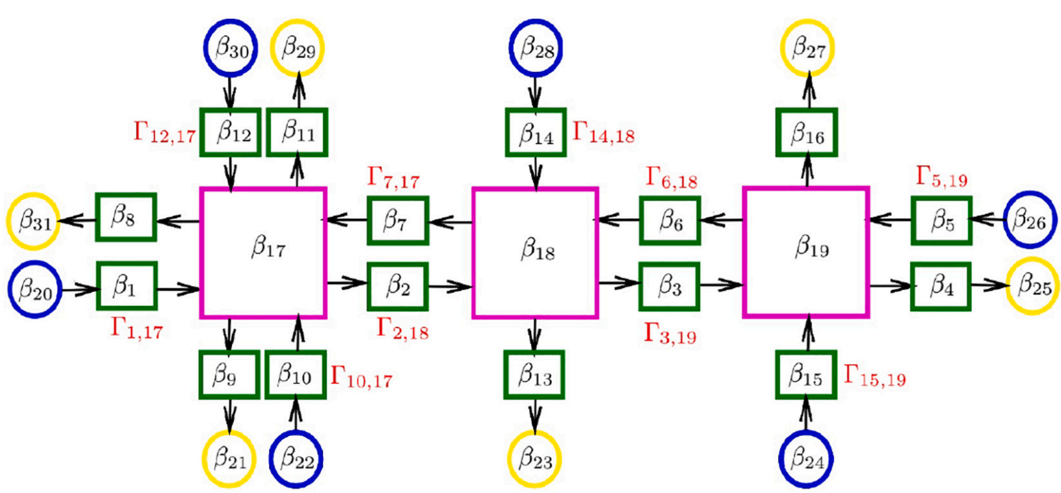


**Fig. 8.** Drawing of the vehicular traffic network.

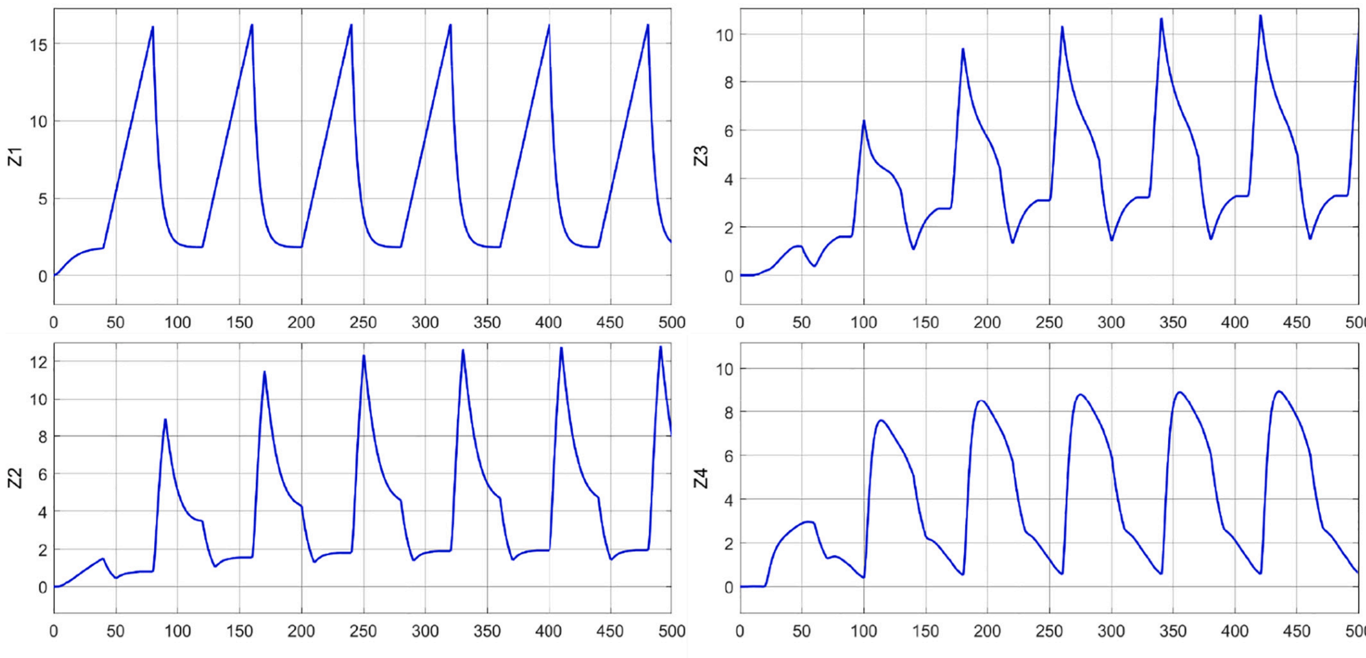


**Fig. 9.** Occupancy simulation response of agents $\beta_1$ to $\beta_4 \in \beta_{st}$.

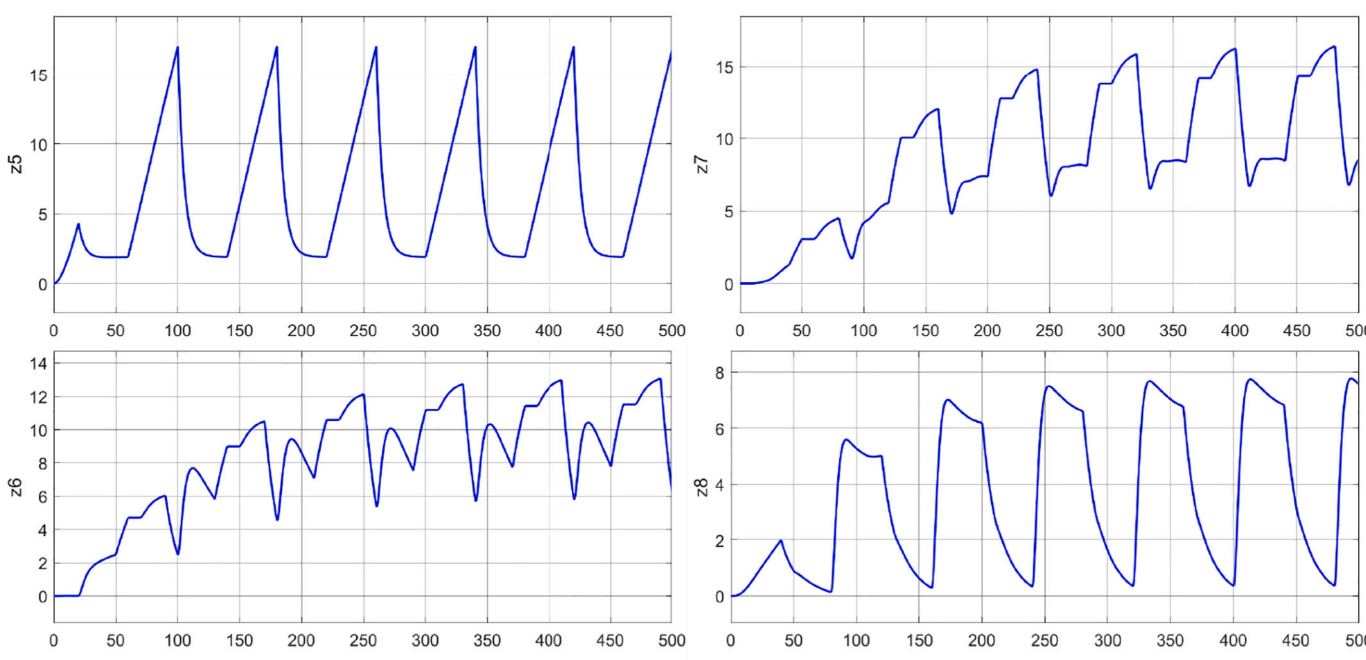


**Fig. 10.** Occupancy simulation response of agents $\beta_5$ to $\beta_8 \in \beta_{st}$.

$$C_{17,9} = [0.06, 0, 0, 0, 0, 0, 0.94, 0, 0, 0, 0, 0]$$

On *8 Octubre and Estero Bellaco*,

$$\Gamma_{18,3} = 0.94\Gamma_{2,18} + 0\Gamma_{6,18} + 0.3\Gamma_{14,18}$$

$$\Gamma_{18,7} = 0.06\Gamma_{2,18} + 1\Gamma_{6,18} + 0.35\Gamma_{14,18}$$

$$\Gamma_{18,13} = 0\Gamma_{2,18} + 0\Gamma_{6,18} + 0.35\Gamma_{14,18}$$

Therefore,

$$C_{18,3} = [0.94, 0, 0, 0, 0, 0, 0.3, 0, 0]$$

$$C_{18,7} = [0.06, 0, 0, 1, 0, 0, 0.35, 0, 0]$$

$$C_{18,13} = [0, 0, 0, 0, 0, 0, 0.35, 0, 0]$$

On 8 Octubre and Joaquín Seccoilla

$$\Gamma_{19,4} = 1\Gamma_{3,19} + 0\Gamma_{5,19} + 0.35\Gamma_{15,19}$$

$$\Gamma_{19,6} = 0\Gamma_{3,19} + 0.94\Gamma_{5,19} + 0.3\Gamma_{15,19}$$

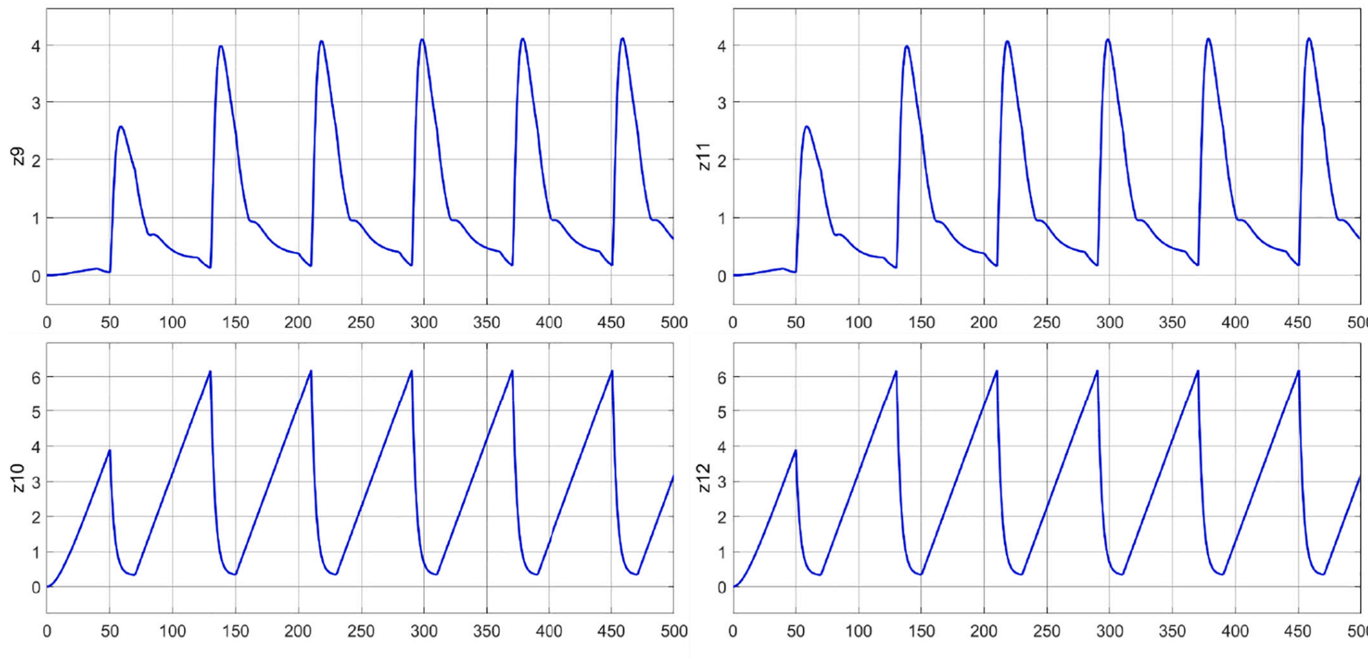


**Fig. 11.** Occupancy of agents $\beta_9$ to $\beta_{12} \in \beta_{st}$.

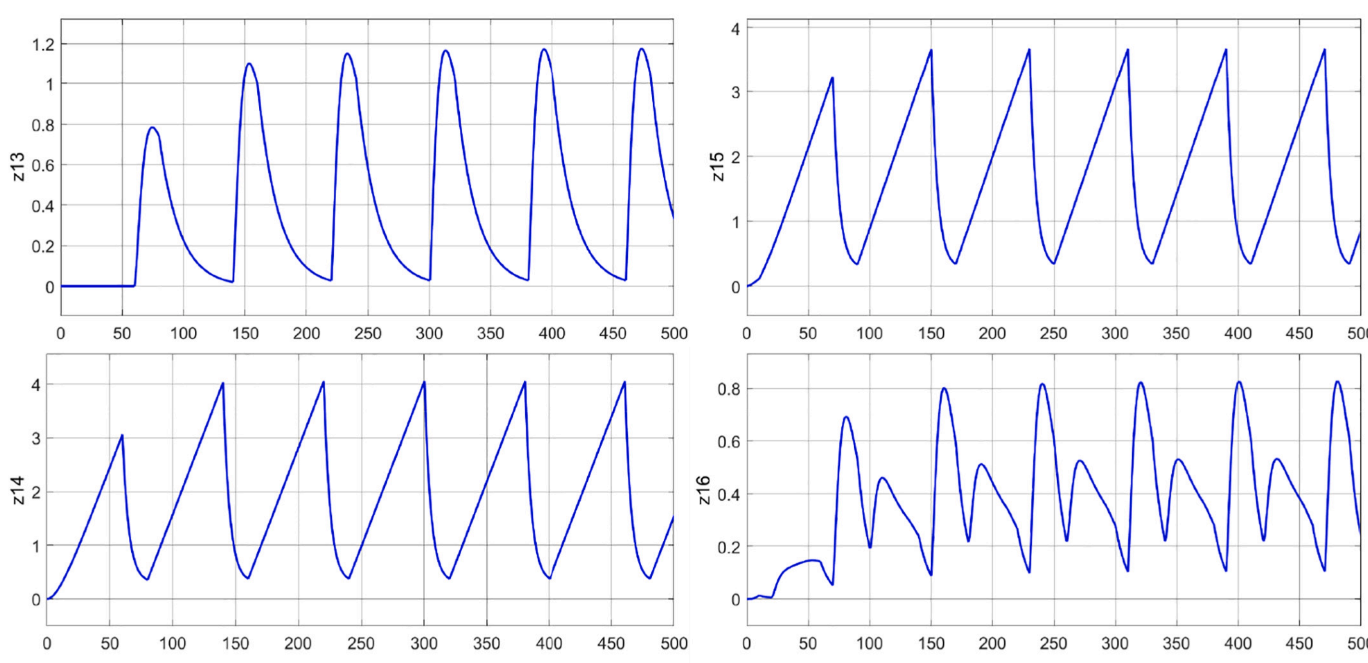


**Fig. 12.** Simulation of $\beta_{13}$ to $\beta_{16} \in \beta_{st}$.

$$\Gamma_{19,16} = 0\Gamma_{3,19} + 0.06\Gamma_{5,19} + 0.35\Gamma_{15,19}$$

Therefore,

$$C_{19,4} = [1, 0, 0, 0, 0, 0, 0.35, 0, 0]$$

$$C_{19,6} = [0, 0, 0, 0.94, 0, 0, 0.3, 0, 0]$$

$$C_{19,16} = [0, 0, 0, 0.06, 0, 0, 0.35, 0, 0]$$

The results of the simulation of agents $\beta_1$ to $\beta_4 \in \beta_{st}$ are shown in the in Fig. 9. In the same way, the occupancy of $\beta_5$ to $\beta_8 \in \beta_{st}$ are shown in the in Fig. 10, Fig. 11 displays the segments of street $\beta_9$ to $\beta_{12} \in \beta_{st}$ and Fig. 12 shows the occupancy of $\beta_{13}$ to $\beta_{16} \in \beta_{st}$.

With the same network data shown above shown in Fig. 7, a simulation was performed in a traffic simulation software package, TSIS-CORSIM. The graphs in Figs. 13 to 16 show the comparison of the data obtained from our model (red color) with respect to that of TSIS-CORSIM(blue color).

Fig. 13 shows the comparison of $\beta_1$ to $\beta_4 \in \beta_{st}$ obtained from the TSIS-CORSIM simulator with respect to our model. In the same way Fig. 14 shows the comparison of $\beta_5$ to $\beta_8 \in \beta_{st}$ for both simulators. The comparison of $\beta_9$ to $\beta_{11} \in \beta_{st}$ is shown in the Fig. 15 and Fig. 16 show $\beta_{13}$ to

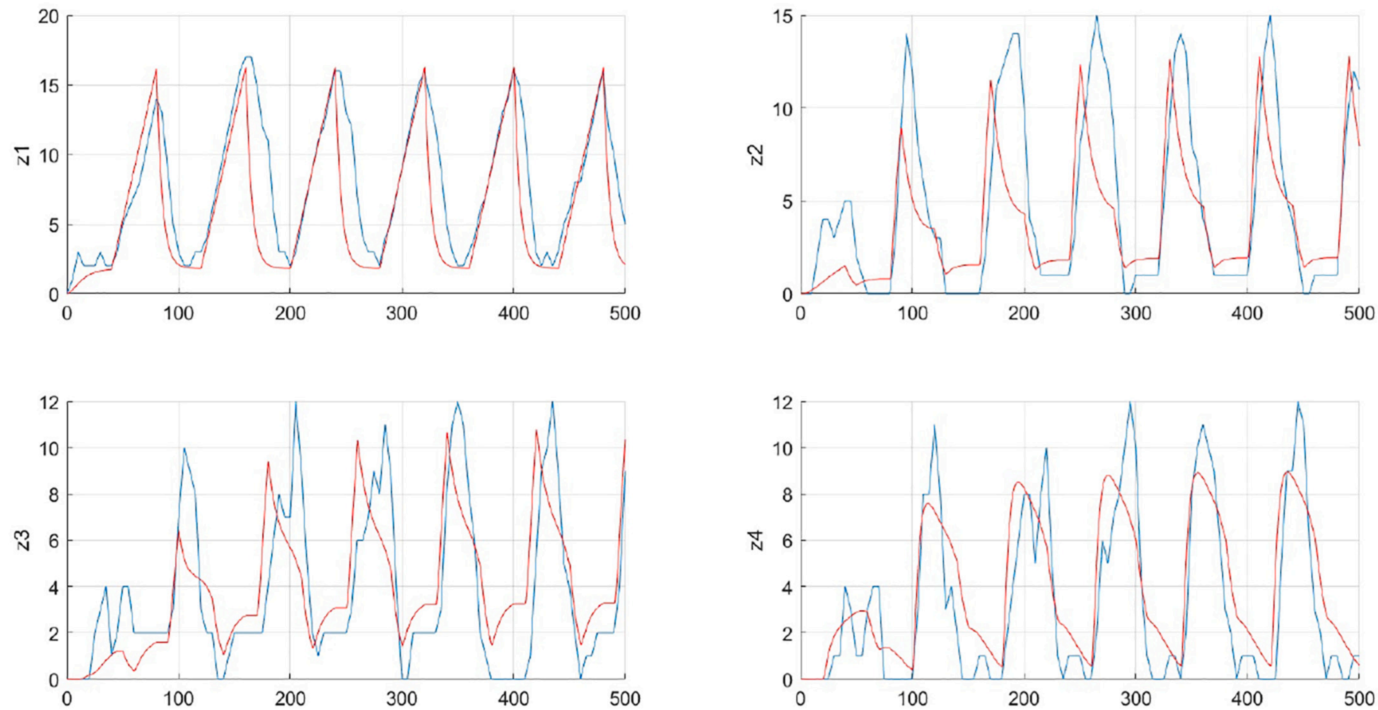


**Fig. 13.** Occupation in agents $\beta_1$ to $\beta_4 \in \beta_{st}$ for both simulators.

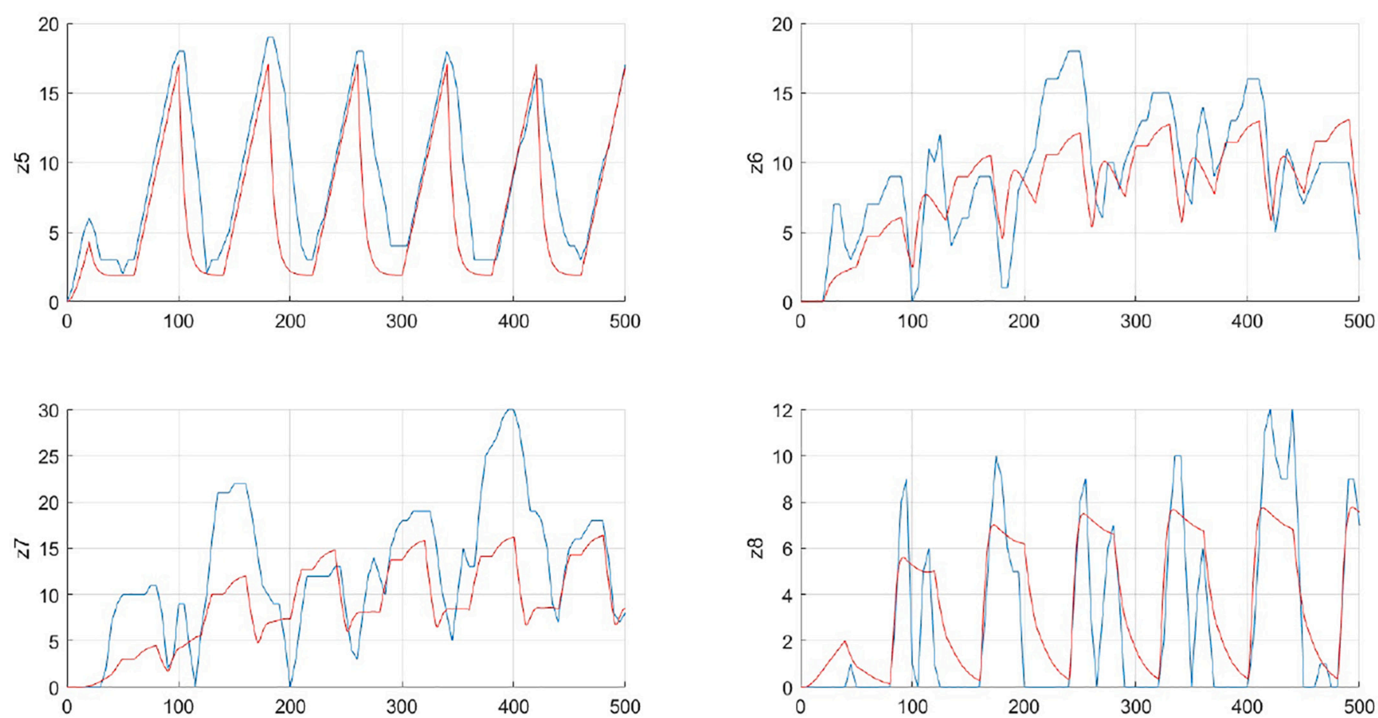


**Fig. 14.** Occupation in agents $\beta_5$ to $\beta_8 \in \beta_{st}$ for both simulators.

$\beta_{16} \in \beta_{st}$.

### 5.2. More complex traffic network 2

A traffic network shown in Fig. 17 was modeled to be applied to a larger network. The initial conditions to this network are: all agents $\beta = \beta_{st} \cup \beta_{cr}$ are initially empty and the maximum occupancy of cars are defined as: $z_i^{\max} = 60$, $i = \{1,3,7,8,11,12,14,17,19,21\}$, $z_j^{\max} = 30$, $j = \{2,4,5,6,9,10,13,15,16,18,20,22\}$. The sources $\beta_{23}$ and $\beta_{29} \in \beta_{so}$ provide 850 and 500 vehicles per hour respectively. The sources $\beta_k \in \beta_{so}$ provide 100 vehicles per hour respectively, $k = \{24,26,30,32\}$. To each intersection $\beta_l \in \beta_{cr}$, $z_l^{\max} = 25$, $l = \{35,36,37,38,39,40,41,42\}$. According to (6) all the crossings in this net are defined in the same way, for example to $\beta_{35} \in \beta_{cr}$, $\Gamma_{35,\,3} = 0.91\Gamma_{1,\,35} + 0.5\Gamma_{2,\,32}$ and $\Gamma_{35,\,4} = 0.09\Gamma_{1,\,35} + 0.5\Gamma_{2,\,32}$.

Therefore, $C_{35,\,3} = [0.91,0,0,0.5,0,0]$ $C_{35,\,4} = [0.09,0,0,0.5,0,0]$.

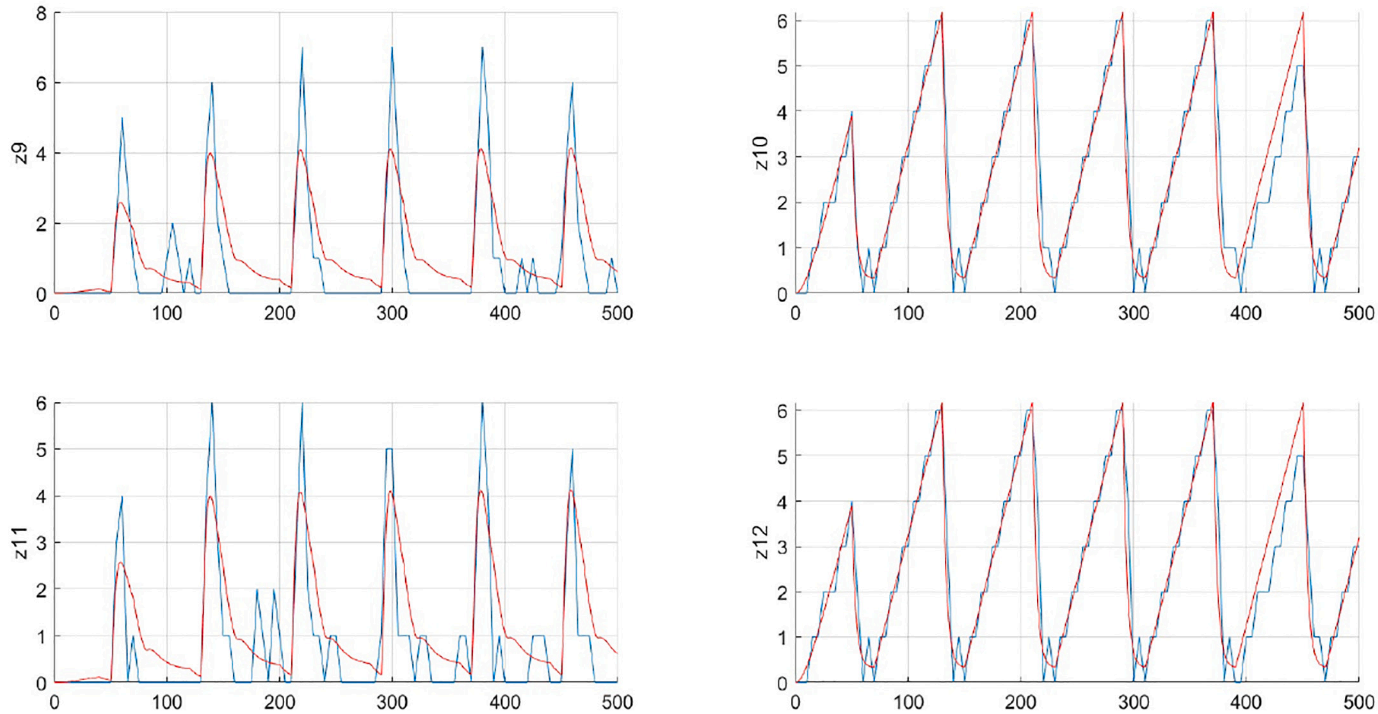


**Fig. 15.** Occupation in agents $\beta_9$ to $\beta_{12} \in \beta_{st}$ for both simulators.

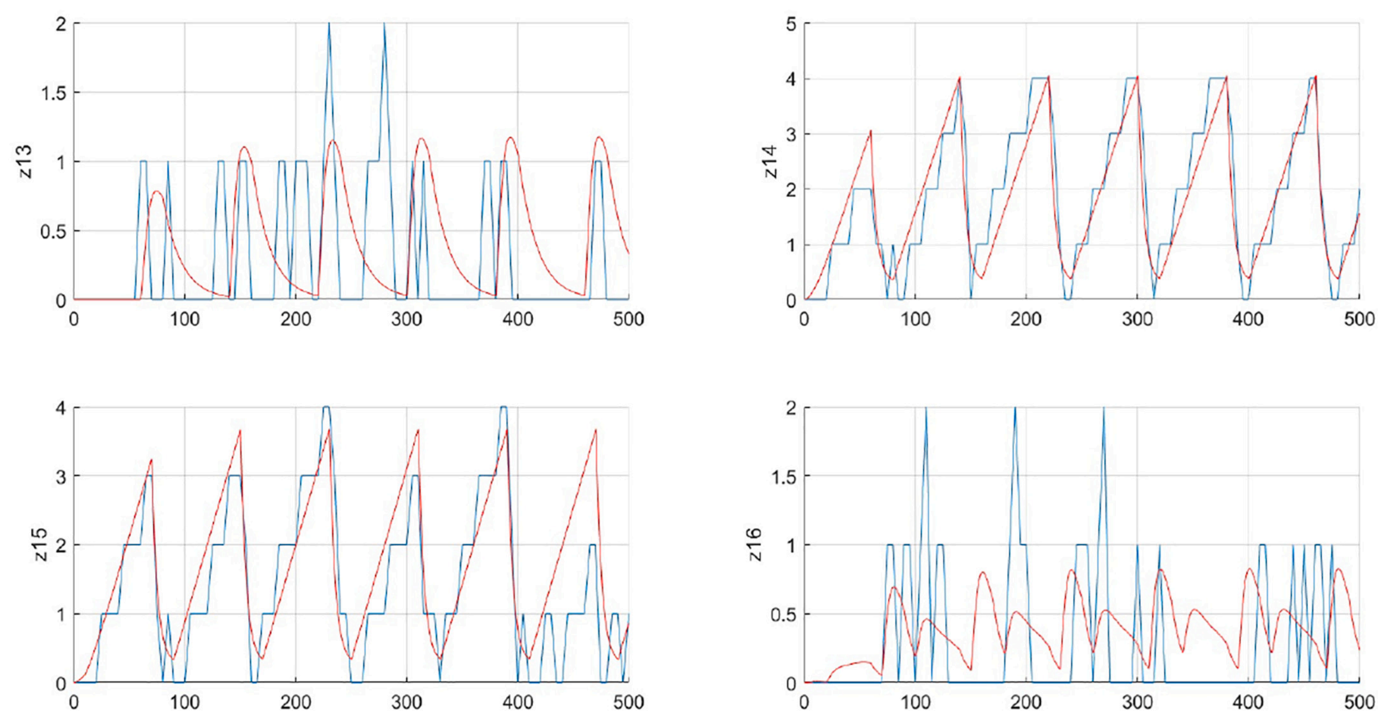


**Fig. 16.** Occupation in agents $\beta_{13}$ to $\beta_{16} \in \beta_{st}$ for both simulators.

The definition of the transitions in each $PN_n$ of this network have a time delay to be fired given by $\delta(t_1) = 40$, $\delta(t_4) = 20$, $\delta(t_2) = \delta(t_5) = 5$ and $\delta(t_3) = \delta(t_6) = 5$.

The simulation results are shown below, to optimize the occupancy in the agents of the vehicular traffic network, two simulations were generated, one with the green wave (GW) effect, applying in the traffic signs associated to a Petri Net and another without the effect of GW. Fig. 18 a) show the occupancy of agents $\beta_1$, $\beta_3$, $\beta_{12}$, $\beta_{14}$, $\beta_{17} \in \beta_{st}$ and Fig. 18 b) the occupancy of segments $\beta_7$, $\beta_8$, $\beta_{11}$, $\beta_{19}$, $\beta_{21} \in \beta_{st}$, both without the GW. Fig. 19 a) and b) shown the results obtained to the same agents of Fig. 18 both with the effect of GW.

To evidence the optimization of flow in the agents, Fig. 20 a) shows the sum of occupancy of agents $\beta_1$, $\beta_3$, $\beta_{12}$, $\beta_{14}$, $\beta_{17} \in \beta_{st}$, in color blue with green wave and in color red without green wave. In the same way, Fig. 20 b) shows the sum of occupancy of agents $\beta_7$, $\beta_8$, $\beta_{11}$, $\beta_{19}$ and $\beta_{21} \in \beta_{st}$, color blue with green wave and color red without. Note that the blue

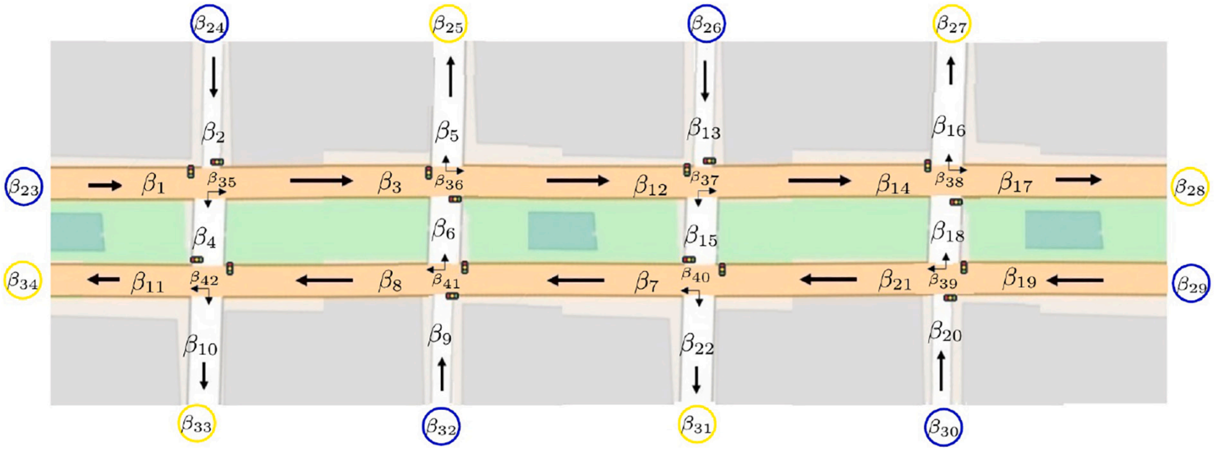


**Fig. 17.** Vehicular network.

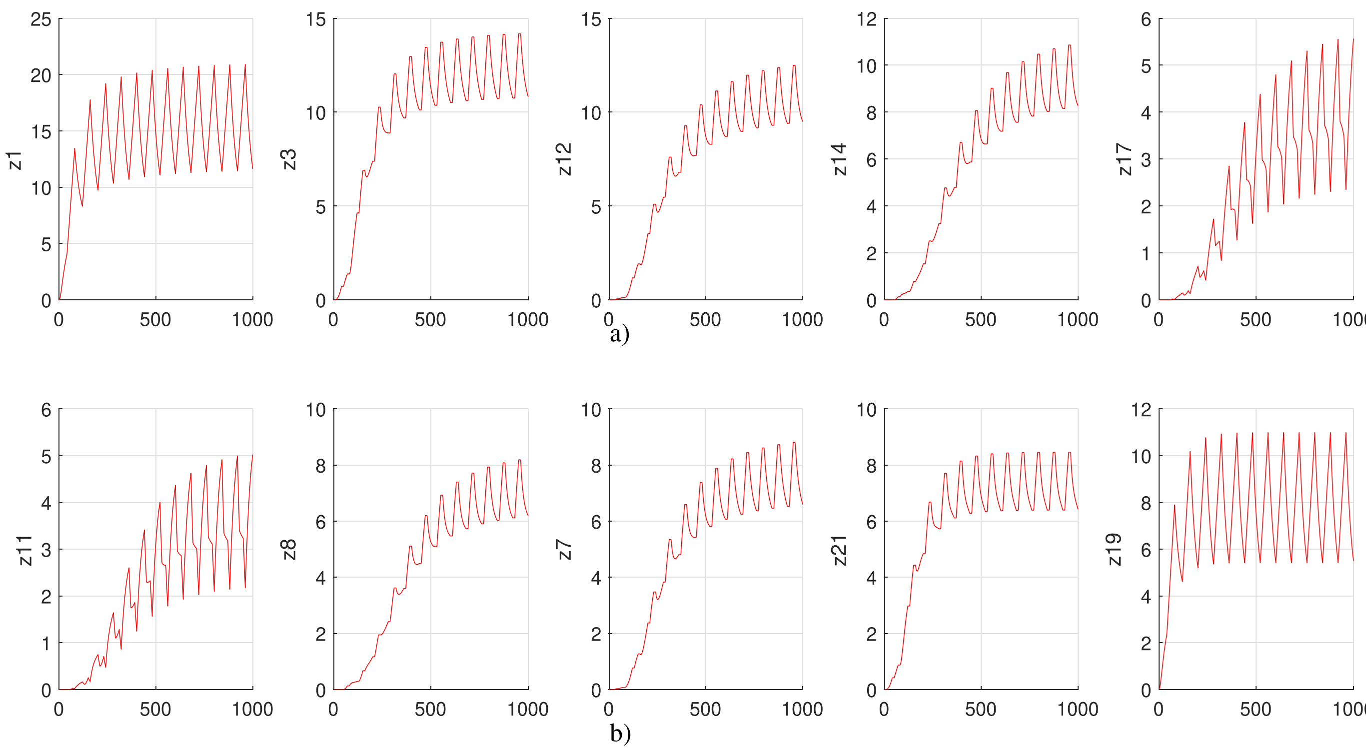


**Fig. 18.** Occupancy of agents without green wave. a) $\beta_1, \beta_3, \beta_{12}, \beta_{14}, \beta_{17} \in \beta_{st}$. b) $\beta_7, \beta_8, \beta_{11}, \beta_{19}, \beta_{21} \in \beta_{st}$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

curves GW show less oscillation compared to those that do not have the effect of GW, as well as the occupation in the agents is less.

## 6. Conclusions and future work

This work proposes the modelling of a complex traffic network as a multi-agent system, where a set of agents related to the dynamics of the density of cars are interconnected. The dynamics of agents is based on the sum of input and output flows of vehicles. The output flows contain the effects of congestion, blocking of flow, traffic lights, etc. The combination of the dynamics of agents can construct the distinct intersections and eventually, a complete network of a urban zone. In order to represent a real effect of the traffic lights, they are modelling as timed Petri Nets, where the times of green, yellow and red phases can be modified easily. Thus, the result is a hybrid representation of continuous flow interacting with a discrete-event system.

The simulation of the hybrid modelling is programmed in Matlab-Simulink. In order to validate the approach, it is compared with a microscopical commercial simulator TSIS-CORSIM addressing a real set of streets and crosses in Montevideo City. The information of flows, length of streets, times of the traffic light phases used is obtained from measured data. The obtained graphics show that the results are quite comparable. It is an important result because the approach avoids the programming of complex simulation as in TSIS-CORSIM, which requires more computational capabilities. Note that the equations of the agents can be easily modified to prove different download rate functions, distributions of flows in an intersection, traffic signal behaviors, etc. Also, a more complex traffic network with distinct streets and intersections is presented where the traffic signals are phase shifted to generate a green wave effect.

Using this multi-agent representation, in further works, will allow to analyze bigger traffic networks, study some strategies of supervisory

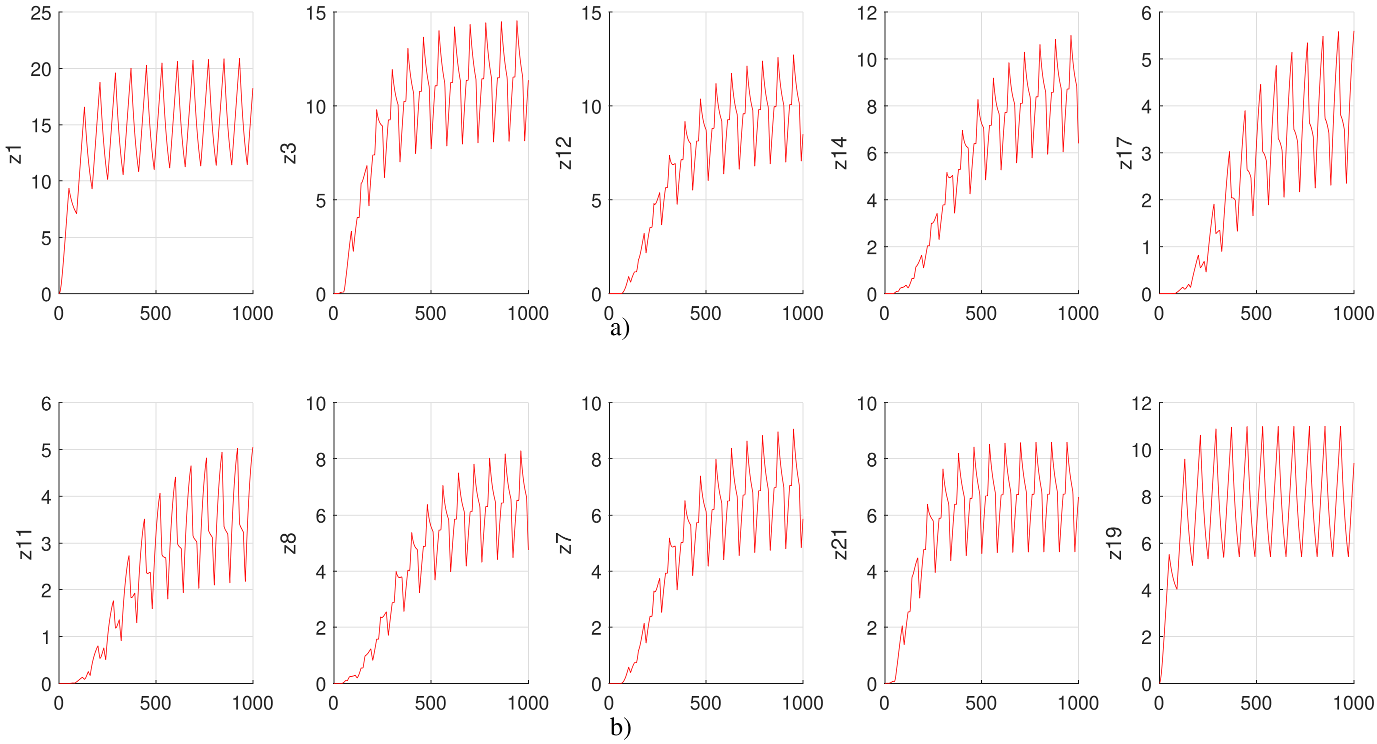


**Fig. 19.** Occupancy of agents with green wave. a) $\beta_1$, $\beta_3$, $\beta_{12}$, $\beta_{14}$, $\beta_{17} \in \beta_{st}$. b) $\beta_7$, $\beta_8$, $\beta_{11}$, $\beta_{19}$, $\beta_{21} \in \beta_{st}$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

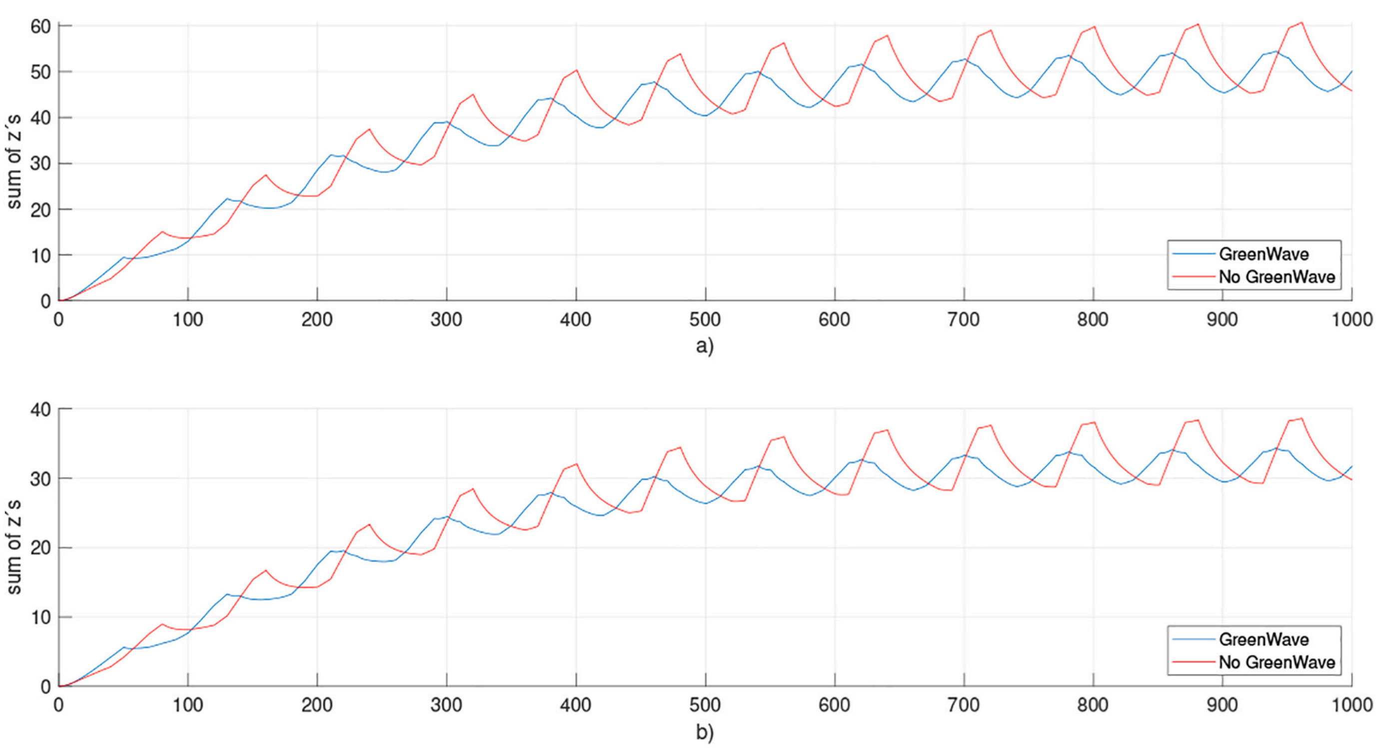


**Fig. 20.** Sum of occupancy of agents. a) $\beta_1$, $\beta_3$, $\beta_{12}$, $\beta_{14}$, $\beta_{17} \in \beta_{st}$. b) $\beta_7$, $\beta_8$, $\beta_{11}$, $\beta_{19}$ and $\beta_{21} \in \beta_{st}$.

control for the timed Petri Nets to improve the flow in a network, and evaluate other types of functions in the dynamics of the agents. Also, some structural properties of the multi-agent system will be addressed based on the nonlinear, hybrid systems and consensus theory.

## Declaration of Competing Interest

None.

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## References

Aboudolas, K., Papageorgiou, M., & Kosmatopoulos, E. (2009). Store-and-forward based methods for the signal control problem in large-scale congested urban road networks. *Transportation Research Part C: Emerging Technologies, 17*(2), 163–174. selected papers from the Sixth Triennial Symposium on Transportation Analysis (TRISTAN VI) https://doi.org/10.1016/j.trc.2008.10.002 http://www.sciencedirect.com/science/article/pii/S0968090X0800082X.

Aboudolas, K., Papageorgiou, M., Kouvelas, A., & Kosmatopoulos, E. (2010). A rolling-horizon quadratic-programming approach to the signal control problem in large-scale congested urban road networks. *Transportation Research Part C: Emerging Technologies, 18*(5), 680–694. applications of Advanced Technologies in Transportation: Selected papers from the 10th AATT Conference https://doi.org/10.1016/j.trc.2009.06.003 http://www.sciencedirect.com/science/article/pii/S0968090X09000813.

Axtell, R. L. (2016). 120 million agents self-organize into 6 million firms: A model of the U.S. private sector. In J. Thangarajah, K. Tuyls, C. Jonker, & S. Marsella (Eds.), *Proceedings of the 2016 International Conference on Autonomous Agents & Multiagent Systems, Richland, SC* (pp. 806–816). International Foundation for Autonomous Agents And Multiagent Systems.

Azzumar, M., Halim, A., & Harjono, M. S. (2013a). Performance evaluation of two ways urban traffic control system based on macroscopic hybrid petri net model. In *2013 International Conference on Advanced Computer Science and Information Systems* (pp. 335–340). https://doi.org/10.1109/ICACSIS.2013.6761598.

Azzumar, M., Halim, A., & Harjono, M. S. (2013b). Performance evaluation of two ways urban traffic control system based on macroscopic hybrid petri net model. In *2013 international conference on advanced computer science and information systems (ICACSIS)* (pp. 335–340). https://doi.org/10.1109/ICACSIS.2013.6761598.

Baldi, S., Michailidis, V. N. I., Kosmatopoulos, E., Papamichail, I., & Papageorgiou, M. (2017). A simulation-based traffic signal control for congested urban traffic networks. *Transportation Science, 53*(1), 1–15. https://doi.org/10.1287/trsc.2017.0754.

Barceló, J. (2010). *Fundamentals of traffic simulation* (1st ed.). New York: Springer.

Basile, F., Chiacchio, P., & Teta, D. (2012). A hybrid model for real time simulation of urban traffic. *Control Engineering Practice, 20*(2), 123–137. https://doi.org/10.1016/j.conengprac.2011.10.002.

Bellemans, T., Schutter, B. D., & Moor, B. D. (2002). Models for traffic control. *Journal A, 43*(3), 13–22.

Borsche, A. M. R. (2019). Microscopic and macroscopic models for coupled car traffic and pedestrian flow. *Journal of Computational and Applied Mathematics, 348*, 356–382. https://doi.org/10.1016/j.cam.2018.08.037.

Chen, B., & Cheng, H. H. (2010). A review of the applications of agent technology in traffic and transportation systems. *IEEE Transactions on Intelligent Transportation Systems, 11*(2), 485–497. https://doi.org/10.1109/TITS.2010.2048313.

Chen, F., Wang, L., Jiang, B., & Wen, C. (2014). A novel hybrid petri net model for urban intersection and its application in signal control strategy. *Journal of the Franklin Institute, 351*(8), 4357–4380. https://doi.org/10.1016/j.jfranklin.2014.05.002.

Chew, Y. R., & Wu, B. S. (2016). A soundscape approach to analyze traffic noise in the city of Taipei, Taiwan. *Computers, Environment and Urban Systems, 59*, 78–85. https://doi.org/10.1016/j.compenvurbsys.2016.05.002. http://www.sciencedirect.com/science/article/pii/S0198971516300680.

Danilevičius, A., & Bogdevičius, M. (2017). Investigation of traffic light switching period affect for traffic flow dynamic processes using discrete model of traffic flow. *Procedia Engineering, 187*, 198–205. https://doi.org/10.1016/j.proeng.2017.04.365. https://www.sciencedirect.com/science/article/pii/S1877705817318957.

DiFebbraro, A., Giglio, D., & Sacco, N. (2016). A deterministic and stochastic petri net model for traffic-responsive signaling control in urban areas. *IEEE Transactions on Intelligent Transportation Systems, 17*(2), 510–524. https://doi.org/10.1109/TITS.2015.2478602.

dos Santos Soares, M., & Vrancken, J. (2012). A modular petri net to modeling and scenario analysis of a network of road traffic signals. *Control Engineering Practice, 20*(11), 1183–1194. https://doi.org/10.1016/j.conengprac.2012.06.005.

Flores-Geronimo, M., Hernandez-Martinez, E. G., Ferreira-Vazquez, E. D., Flores-Godoy, J. J., & Fernandez-Anaya, G. (2019). A hybrid representation of urban traffic networks using multi-agent systems and petri nets. In *2019 6th International Conference on Control, Decision and Information Technologies (CoDIT)* (pp. 1562–1567). https://doi.org/10.1109/CoDIT.2019.8820626.

Flores-Geronimo, M., Hernandez-Martinez, E. G., Ferreira-Vazquez, E. D., Flores-Godoy, J. J., & Fernandez-Anaya, G. (2018). Modular modelling for urban traffic networks based on multi-agent systems and petri nets. In *2018 XX Congreso Mexicano de Robótica (COMRob)* (pp. 1–7). https://doi.org/10.1109/COMROB.2018.8689413.

Goñi-Ros, B., Knoop, V. L., Takahashi, T., Sakata, I., van Arem, B., & Hoogendoorn, S. P. (2016). Optimization of traffic flow at freeway sags by controlling the acceleration of vehicles equipped with in-car systems. *Transportation Research Part C: Emerging Technologies, 71*, 1–18. https://doi.org/10.1016/j.trc.2016.06.022. http://www.sciencedirect.com/science/article/pii/S0968090X16300973.

Grote, M., Williams, I., Preston, J., & Kemp, S. (2016). Including congestion effects in urban road traffic CO2 emissions modelling: Do local government authorities have the right options? *Transportation Research Part D: Transport and Environment, 43*, 95–106. https://doi.org/10.1016/j.trd.2015.12.010.

Hoogendoorn, S. P., & Bovy, P. H. L. (2001). State-of-the-art of vehicular traffic flow modelling. *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering, 215*(4), 283–303. https://doi.org/10.1177/095965180121500402.

Hossain, M., & McDonald, P. (1998). Modelling of traffic operations in urban networks of developing countries: a computer aided simulation approach. *Computers, Environment and Urban Systems, 22*(5), 465–483. https://doi.org/10.1016/S0198-9715(98)00040-4. http://www.sciencedirect.com/science/article/pii/S0198971598000404.

I. N. de Estadística y Geografía. (2017). *Encuesta origen destino en hogares de la zona metropolitana del valle de méxico eod 2017*. Tech. rep. Instituto Nacional de Estadística y Geografía.

Jia, S., Yan, G., & Shen, A. (2018). Traffic and emissions impact of the combination scenarios of air pollution charging fee and subsidy. *Journal of Cleaner Production, 197*, 678–689. https://doi.org/10.1016/j.jclepro.2018.06.117.

Jiang, G.-X., Chen, Y., Zhang, L.-G., & Li, H. (2009). Modeling and reachability analysis for single intersection based on rectangular hybrid automata. *Journal of Transportation Systems Engineering and Information Technology, 9*(4), 120–126.

Kawasaki, Y., Hara, Y., & Kuwahara, M. (2017). Real-time monitoring of dynamic traffic states by state-space model. *Transportation Research Procedia, 21*, 42–55. https://doi.org/10.1016/j.trpro.2017.03.076.

Klawtanong, M., & Limkumnerd, S. (2019). Dissipation of traffic congestion using autonomous-based car-following model with modified optimal velocity. *Physica A: Statistical Mechanics and its Applications*. https://doi.org/10.1016/j.physa.2019.123412. http://www.sciencedirect.com/science/article/pii/S0378437119319065.

Kohan, M., & Ale, J. M. (2020). Discovering traffic congestion through traffic flow patterns generated by moving object trajectories. *Computers, Environment and Urban Systems, 80*, Article 101426. https://doi.org/10.1016/j.compenvurbsys.2019.101426. http://www.sciencedirect.com/science/article/pii/S0198971518305684.

Kong, Q., Tu, S., & Liu, Y. (2012). A traffic-network-model-based algorithm for short-term prediction of urban traffic flow. In *Proceedings of 2012 IEEE international conference on service operations and logistics, and informatics* (pp. 129–132). https://doi.org/10.1109/SOLI.2012.6273517.

Liu, D., Baldi, S., Yu, W., & Chen, G. (2020). On distributed implementation of switch-based adaptive dynamic programming. *IEEE Transactions on Cybernetics*, 1–7. https://doi.org/10.1109/TCYB.2020.3029825.

Liu, D., Yu, W., Baldi, S., Cao, J., & Huang, W. (2020). A switching-based adaptive dynamic programming method to optimal traffic signaling. *IEEE Transactions on Systems, Man, and Cybernetics: Systems, 50*(11), 4160–4170. https://doi.org/10.1109/TSMC.2019.2930138.

Maecki, K., & Iwan, S. (2019). Modeling traffic flow on two-lane roads with traffic lights and countdown timer. *Transportation Research Procedia, 39*, 300–308, 3rd International Conference Green Cities – Green Logistics for Greener Cities, Szczecin, 13–14 September 2018.

Manley, E., Cheng, T., Penn, A., & Emmonds, A. (2014). A framework for simulating large-scale complex urban traffic dynamics through hybrid agent-based modelling. *Computers, Environment and Urban Systems, 44*, 27–36. https://doi.org/10.1016/j.compenvurbsys.2013.11.003. http://www.sciencedirect.com/science/article/pii/S0198971513001129.

Nagatani, T. (2018). Effect of bypasses on vehicular traffic through a series of signals. *Physica A: Statistical Mechanics and its Applications, 506*, 229–236. https://doi.org/10.1016/j.physa.2018.04.027. http://www.sciencedirect.com/science/article/pii/S0378437118304606.

Nagatani, T., & Tobita, K. (2012). Vehicular motion in counter traffic flow through a series of signals controlled by a phase shift. *Physica A: Statistical Mechanics and its Applications, 391*(20), 4976–4985. https://doi.org/10.1016/j.physa.2012.05.044. http://www.sciencedirect.com/science/article/pii/S0378437112004153.

Nega, T., Smith, C., Bethune, J., & Fu, W.-H. (2012). An analysis of landscape penetration by road infrastructure and traffic noise. *Computers, Environment and Urban Systems, 36*(3), 245–256. https://doi.org/10.1016/j.compenvurbsys.2011.09.001. http://www.sciencedirect.com/science/article/pii/S0198971511000895.

Ng, K. M., Reaz, M. B. I., & Ali, M. A. M. (2013). A review on the applications of petri nets in modeling, analysis, and control of urban traffic. *IEEE Transactions on Intelligent Transportation Systems, 14*(2), 858–870. https://doi.org/10.1109/TITS.2013.2246153.

Otero, D., Galetti, D., & Mizrahi, S. S. (2018). Modeling vehicular traffic networks. Part i. *Physica A: Statistical Mechanics and its Applications, 509*, 97–110. https://doi.org/10.1016/j.physa.2018.06.016. http://www.sciencedirect.com/science/article/pii/S0378437118307404.

Pahnabi, A., Karimpour, A., & Mobara, M. (2016). Control of urban traffic network based on mixed logical dynamical modeling and constrained predictive control approach. *International Journal of Scientific Engineering and Technology, 5*(6), 367–371.

Qi, L., Zhou, M., & Luan, W. (2015). Modeling and control of urban road intersections with incidents via timed petri nets. In *2015 IEEE 12th international conference on networking, sensing and control* (pp. 185–190). https://doi.org/10.1109/ICNSC.2015.7116032.

Renato Vázquez, C., Sutarto, H. Y., Boel, R., & Silva, M. (2010). Hybrid petri net model of a traffic intersection in an urban network. In *2010 IEEE international conference on control applications* (pp. 658–664). https://doi.org/10.1109/CCA.2010.5611322.

Sharma, A. R., Kharol, S. K., & Badarinath, K. (2010). Influence of vehicular traffic on urban air quality – A case study of Hyderabad, India. *Transportation Research Part D:*

*Transport and Environment, 15*(3), 154–159. https://doi.org/10.1016/j.trd.2009.11.001.

Silva, S. D., Ball, A. S., Huynh, T., & Reichman, S. M. (2016). Metal accumulation in roadside soil in Melbourne, Australia: Effect of road age, traffic density and vehicular speed Special Issue: Urban Health and Wellbeing. *Environmental Pollution, 208*, 102–109. https://doi.org/10.1016/j.envpol.2015.09.032.

Teng, F., Zhang, H., Luo, C., & Shan, Q. (2019). Delay tolerant containment control for second-order multi-agent systems based on communication topology design. *Neurocomputing*. https://doi.org/10.1016/j.neucom.2019.10.001. http://www.sciencedirect.com/science/article/pii/S0925231219313736.

Tolba, C., Lefebvre, D., Thomas, P., & Moudni, A. E. (2005). Continuous and timed petri nets for the macroscopic and microscopic traffic flow modeling. *Simulation Modeling Practice and Theory, 13*(5), 407–436. https://doi.org/10.1016/j.simpat.2005.01.001.

van Wageningen-Kessels, F., van Lint, H., Vuik, K., & Hoogendoorn, S. (2015a). Genealogy of traffic flow models. *EURO Journal on Transportation and Logistics, 4*(4), 445–473. https://doi.org/10.1007/s13676-014-0045-5.

van Wageningen-Kessels, F., van Lint, H., Vuik, K., & Hoogendoorn, S. (2015b). Genealogy of traffic flow models. *EURO Journal on Transportation and Logistics, 4*(4), 445–473. https://doi.org/10.1007/s13676-014-0045-5.

Wang, L., Dai, L., Xiao-ming, L., & Zheng-xi, L. (2013). Regional traffic state consensus optimization based on computational experiments. In *16th International IEEE conference on intelligent transportation systems (ITSC 2013)* (pp. 1547–1552). https://doi.org/10.1109/ITSC.2013.6728450.

Wang, Y., Cao, J., Wang, H., & Alsaadi, F. E. (2019). Event-triggered consensus of multi-agent systems with nonlinear dynamics and communication delay. *Physica A: Statistical Mechanics and its Applications, 522*, 147–157. https://doi.org/10.1016/j.physa.2019.01.124. http://www.sciencedirect.com/science/article/pii/S0378437119301311.

Wang, Z., & Zlatanova, S. (2016). Multi-agent based path planning for first responders among moving obstacles. *Computers, Environment and Urban Systems, 56*, 48–58. https://doi.org/10.1016/j.compenvurbsys.2015.11.001. http://www.sciencedirect.com/science/article/pii/S0198971515300314.

Wang, Z., Li, Y., & Cai, P. (2012). Supervisory arc of petri nets and its application on traffic signals control system. In *International conference on automatic control and artificial intelligence (ACAI 2012)* (pp. 2013–2018). https://doi.org/10.1049/cp.2012.1391.

Wey, W.-M. (2000). Model formulation and solution algorithm of traffic signal control in an urban network. *Computers, Environment and Urban Systems, 24*(4), 355–378. https://doi.org/10.1016/S0198-9715(00)00002-8. http://www.sciencedirect.com/science/article/pii/S0198971500000028.

Xie, B., Xu, M., Härri, J., & Chen, Y. (2013). A traffic light extension to cell transmission model for estimating urban traffic jam. In *2013 IEEE 24th annual international symposium on personal, indoor, and mobile radio communications (PIMRC)* (pp. 2566–2570). https://doi.org/10.1109/PIMRC.2013.6666579.

Xie, R., Wei, D., Han, F., Lu, Y., Fanga, J., Liu, Y., & Wang, J. (2019). The effect of traffic density on smog pollution: Evidence from chinese cities. *Technological Forecasting & Social Change, 144*, 421–427. https://doi.org/10.1016/j.techfore.2018.04.023.

Yang, Y., Zhang, X.-M., He, W., Han, Q.-L., & Peng, C. (2020). Sampled-position states based consensus of networked multi-agent systems with second-order dynamics subject to communication delays. *Information Sciences, 509*, 36–46. https://doi.org/10.1016/j.ins.2019.08.073. http://www.sciencedirect.com/science/article/pii/S0020025519308254.

Yanga, M., Mab, T., & Sunb, C. (2018). Evaluating the impact of urban traffic investment on SO2 emissions in china cities. *Energy Policy, 113*, 20–27. https://doi.org/10.1016/j.enpol.2017.10.039.

Yolton, K., Khoury, J. C., Burkle, J., LeMasters, G., Cecil, K., & Ryan, P. (2019). Lifetime exposure to traffic-related air pollution and symptoms of depression and anxiety at age 12 years. *Environmental Research, 173*, 199–206. https://doi.org/10.1016/j.envres.2019.03.005.

Yu, Y., Kamel, A. E., Gong, G., & Li, F. (2014). Multi-agent based modeling and simulation of microscopic traffic in virtual reality system. *Simulation Modelling Practice and Theory, 45*, 62–79. https://doi.org/10.1016/j.simpat.2014.04.001.

Yuan, Y., Duret, A., & van Lint, H. (2015). Mesoscopic traffic state estimation based on a variational formulation of the lwr model in lagrangian-space coordinates and kalman filter. *Transportation Research Procedia, 10*, 82–92. https://doi.org/10.1016/j.trpro.2015.09.058.

Zhang, Y., Xue, Y., Zhang, P., Fan, D., & di He, H. (2019). Bifurcation analysis of traffic flow through an improved car-following model considering the time-delayed velocity difference. *Physica A: Statistical Mechanics and its Applications, 514*, 133–140. https://doi.org/10.1016/j.physa.2018.09.012. http://www.sciencedirect.com/science/article/pii/S0378437118311373.

Zhonghe, H., Chi, Z., & Li, W. (2015). Consensus feedback control for urban road traffic networks. In *2015 54th annual conference of the society of instrument and control engineers of Japan (SICE)* (pp. 1413–1418). https://doi.org/10.1109/SICE.2015.7285401.

Zhou, Z., De Schutter, B., Lin, S., & Xi, Y. (2017). Two-level hierarchical model-based predictive control for large-scale urban traffic networks. *IEEE Transactions on Control Systems Technology, 25*(2), 496–508. https://doi.org/10.1109/TCST.2016.2572169.